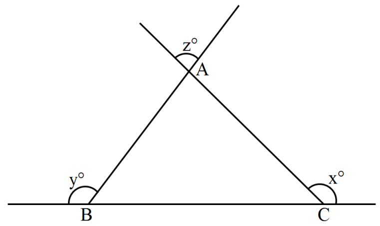
KD EDUCATION ACADEMY [9582701166]

Time : 6 hour | STD 9 Maths | Total Marks : 300

kd sir 90+ Questions ch-6 triangle

* Choose the right answer from the given options. [1 Marks Each] [73]

1. In figure, what is z in terms of x and y?



(A) 180º - (x + y) (B) x + y + 180º (C) x + y + 360º (D) x + y - 180º

Ans. :

d. x + y - 180º

Solution:

From figure

$\angle \mathrm{A} = \mathrm{z}^{\circ}$

$\angle \mathrm{ACB} = 180 - \mathrm{z}^{\circ}$

$\angle \mathrm{ABC} = 180 - \angle \mathrm{y}^{\circ}$

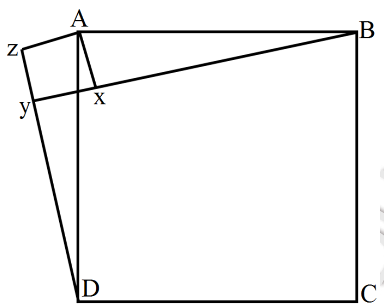
Now, in $\triangle \mathrm{ABC}$

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^{\circ}$

$\Rightarrow \mathrm{z}^{\circ} + 180 - \mathrm{y}^{\circ} + 180^{\circ} - \mathrm{x}^{\circ} = 180^{\circ}$

$\Rightarrow \mathrm{z}^{\circ} = \mathrm{x}^{\circ} + \mathrm{y}^{\circ} - 180^{\circ}$

2. In figure, X is a point in the interior of square ABCD. AXYZ is also a square. If DY = 3cm and AZ = 2cm, then BY =



(A) 6cm (B) 7cm (C) 8cm (D) 5cm

Ans. :

b. 7cm

Solution:

$\angle \mathrm{Z} = 90^{\circ}$ (Angle of square)

Therefore, AZD is a right angle triangle,

By Pythagoras theorem,

$AD^2 = AZ^2 + ZD^2$

$AD^2 = 22 + (2 + 3)^2$

$AD^2 = 4 + 25$

$\mathrm{AD} = \sqrt{29}$

In $\triangle \mathrm{AXB}$, with X as right angle,

By Pythagoras theorem,

$AB^2 = AX^2 + XB^2$

$XB^2 = 29 - 4$

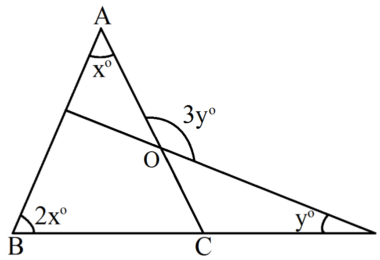
XB = 5

BY = XB + XY

= 5 + 2

= 7cm

3. In Fig. what is y in terms of x?

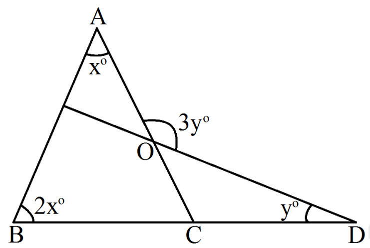


(A) $\frac{3}{2}\mathrm{x}^\circ$ (B) $\frac{4}{3}\mathrm{x}^\circ$ (C) x° (D) $\frac{3}{4}\mathrm{x}^\circ$

Ans. :

a. $\frac{3}{2}\mathrm{x}^\circ$

Solution:



From figure,

$\angle\mathrm{DOC} = 180^\circ - \angle\mathrm{AOD}$ (Both are Supplementary)

$\Rightarrow \angle\mathrm{DOC} = 180^\circ - 3\mathrm{y}^\circ$

Also, $\angle\mathrm{ACB} = 180^\circ - \angle\mathrm{A} - \angle\mathrm{B}$

$\Rightarrow \angle\mathrm{ACB} = 180^\circ - \mathrm{x}^\circ - 2\mathrm{x}^\circ = 180^\circ - 3\mathrm{x}^\circ$

And $\angle\mathrm{ACD} = 180^\circ - \angle\mathrm{ACB}$

$= 180^\circ - (180^\circ - 3\mathrm{x}^\circ)$

$\Rightarrow \angle\mathrm{ACD} = 3\mathrm{x}^\circ$

Now, in $\triangle\mathrm{OCD}$

$\angle\mathrm{DOC} + \angle\mathrm{OCD} + \angle\mathrm{D} = 180^\circ$

$180^\circ - 3\mathrm{y}^\circ + 3\mathrm{x}^\circ + \mathrm{y}^\circ = 180^\circ \ [\angle\mathrm{OCD} = \angle\mathrm{ACD}]$

$\Rightarrow 2\mathrm{y}^\circ = 3\mathrm{x}^\circ$

$\Rightarrow \mathrm{y} = \frac{3}{2}\mathrm{x}^\circ$

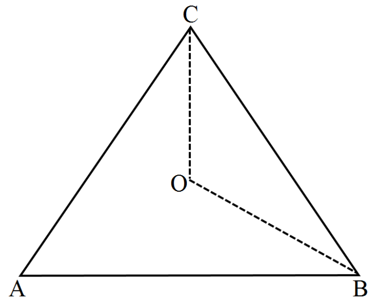
4. In a $\triangle \mathrm{ABC}$, if $\angle \mathrm{A}=60^{\circ},\ \angle \mathrm{B}=80^{\circ}$ and the bisectors of $\angle \mathrm{B}$ and $\angle \mathrm{C}$ meet at O, the $\angle \mathrm{BOC}=$

(A) 120º (B) 150º (C) 30º (D) 60º

Ans. :

a. 120º

Solution:



O is point where bisectors of $\angle \mathrm{C}$ & $\angle \mathrm{B}$ meets.

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}$

$60^{\circ}+80^{\circ}+\angle \mathrm{C}=180^{\circ}$

$\angle \mathrm{C}=40^{\circ}$

$\angle \mathrm{C} 2=20^{\circ}$

$\angle \mathrm{C} 2=20^{\circ}=\angle \mathrm{BCO} \ldots$ (i)

$\angle \mathrm{B} 2=80^{\circ}=40^{\circ}=\angle \mathrm{OBC} \ldots$ (ii)

In $\triangle \mathrm{BOC}$

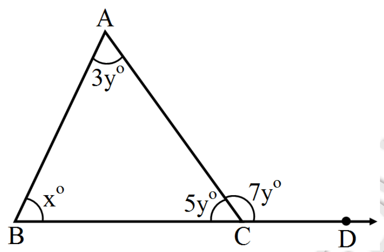
$\angle \mathrm{BCO}+\angle \mathrm{OBC}+\angle \mathrm{BOC}=180^{\circ}$

From (i) and (ii)

$20^{\circ}+40^{\circ}+\angle \mathrm{BOC}=180^{\circ}$

$\angle \mathrm{BOC}=180^{\circ}-60^{\circ}=120^{\circ}$

5. In the given figure, side BC of $\triangle \mathrm{ABC}$ has been produced to a point D. If $\angle \mathrm{A}=3 \mathrm{y}, \angle \mathrm{B}=\mathrm{x}^{\circ}, \angle \mathrm{C}=5 \mathrm{y}^{\circ}$ and $\angle \mathrm{CBD}=7 \mathrm{y}^{\circ}$. Then, the value of x is:



(A) 60 (B) 50 (C) 45 (D) 35

Ans. :

a. 60

Solution:

$\angle \mathrm{ACB}=\angle \mathrm{ACD}=180^{\circ}$ (linear pair)

$\Rightarrow 5 \mathrm{y}+7 \mathrm{y}=180^{\circ}$

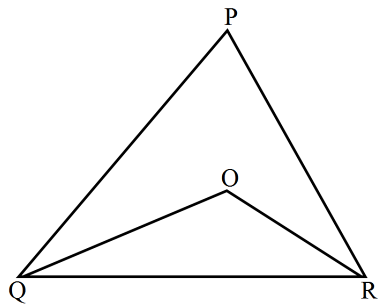
$\Rightarrow 12 \mathrm{y}=180^{\circ}$

$\Rightarrow \mathrm{y}=15^{\circ}$

Now, $\angle \mathrm{ACD}=\angle \mathrm{ABC}+\angle \mathrm{BAC}$ (Exterior angle property)

$$\Rightarrow 7y = x + 3y$$
$$\Rightarrow 7(15^\circ) = x + 3(15^\circ)$$
$$\Rightarrow 105^\circ = x + 45^\circ$$
$$\Rightarrow x = 60^\circ$$

6. In the adjoining figure, PQ > PR. If OQ and OR are bisectors of $\angle Q$ and $\angle R$ respectively, then



(A) OQ > OR (B) OQ < OR (C) OQ ≤ OR (D) OQ = OR

Ans. :

a. OQ > OR

Solution:

Since PQ > PR then $\angle R > \angle Q$ and hence their bisectors follow the same.

I.e $\frac{R}{2} > \frac{Q}{2}$ and hence OQ > OR.

7. An angle is 14º more than its complement. Find its measure.

(A) 52º (B) 62º (C) 32º (D) 42º

Ans. :

a. 52º

Solution:

Let the angle = x

Its complement = 90º - x

According to the question, x is 14° more than its complement,

⇒ x = (90° - x) + 14°

⇒ x + x = 104°

⇒ 2x = 104°

$$\Rightarrow \ X = \frac{104^\circ}{2} = 52^\circ$$

8. The perimeter of a triangle is 36cm and its sides are in the ratio a : b : c = 3 : 4 : 5 then a, b, c are respectively:

(A) 9cm, 15cm, 12cm (B) 9cm, 12cm, 15cm (C) 12cm, 9cm, 15cm (D) 15cm, 12cm, 9cm

Ans. :

b. 9cm, 12cm, 15cm

Solution:

Let the three sides a, b, c be 3x, 4x and 5x respectively.

Then according to the conditions given in the question, we have

3x + 4x + 5x = 36

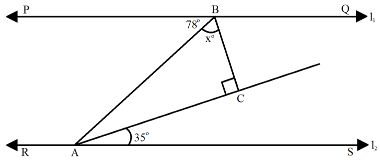
12x = 36

x = 3cm

Thus, the three sides are:

a = 3 × 3 = 9cm, b = 4 × 3 = 12cm and c = 5 × 3 = 15cm

9. In Fig. for which value of x is $l_1 \parallel l_2$?



(A) 37º (B) 43º (C) 45º (D) 47º

Ans. :

d. 47º

Solution:

Let if $l_1 \parallel l_2$ and AB is tranverse to it.

Then,

$\angle \mathrm{PBA}$ should be equal $\angle \mathrm{BAS}$ (Alternate angles)

So if $l_1 \parallel l_2$, then $\angle \mathrm{BAS} = 70^\circ$

$\Rightarrow \angle \mathrm{BAC} = 78^\circ - 35^\circ = 43^\circ \ldots (1)$

Now, in $\triangle \mathrm{ABC}$

$\mathrm{x}^\circ + \angle \mathrm{C} + \angle \mathrm{BAC} = 180^\circ$

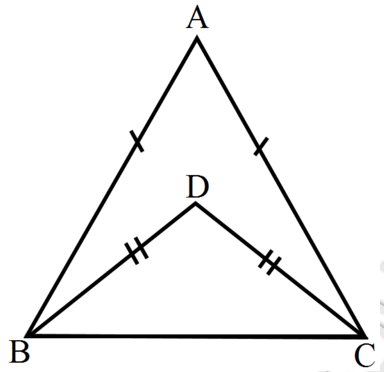
$\Rightarrow \mathrm{x}^\circ + 90^\circ + 43^\circ = 180^\circ$

$\Rightarrow \mathrm{x}^\circ = 180^\circ - 90^\circ - 43^\circ = 47^\circ$

$\Rightarrow \mathrm{x}^\circ = 47^\circ$

So if xº = 47º then $l_1 \parallel l_2$

10. In the adjoining Figure, AB = AC and BO = CD. The ratio $\angle \mathrm{ABO} : \angle \mathrm{ACD}$ is:



(A) It is 1 : 1 (B) It is 1 : 2 (C) It is 2 : 3 (D) It is 2 : 1

Ans. :

a. It is 1 : 1

Solution:

In $\triangle \mathrm{ABC}$

$\mathrm{AB} = \mathrm{AC}$

$\therefore \angle \mathrm{ABC} = \angle \mathrm{ACB}$ (angles opposite to equal sides of a triangle are equal) ...(1)

In $\triangle \mathrm{DBC}$,

$\mathrm{DB} = \mathrm{DC}$,

$\therefore \angle\mathrm{DBC} = \angle\mathrm{DCB}$ (angles opposite to equal sides of a triangle are equal) ...(2)

subtract 2 from 1

$\angle\mathrm{ABC} - \angle\mathrm{DBC} = \angle\mathrm{ACB} - \angle\mathrm{DCB}$ (equals subtracted from equals gives equal)

$= \angle\mathrm{ABD} = \angle\mathrm{ACD}$

Divide both the sides by $\triangle\mathrm{ACD}$

$\Rightarrow \frac{\angle\mathrm{ABD}}{\angle\mathrm{ACD}} = 1$

$\therefore \angle\mathrm{ABD} : \angle\mathrm{ACD} = 1 : 1$

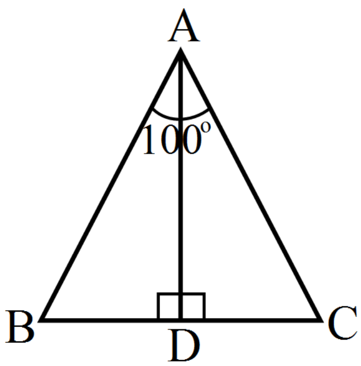
11. In $\triangle\mathrm{ABC}$, if $\angle\mathrm{A} = 100^\circ, \mathrm{AD}$ bisects $\angle\mathrm{A}$ and $\mathrm{AD} \perp \mathrm{BC}$. Then, $\angle\mathrm{B} =$

(A) 50º (B) 90º (C) 40º (D) 100º

Ans. :

c. 40º

Solution:



$\mathrm{AD} \perp \mathrm{BC}$ and AD bisects $\angle\mathrm{A}$.

$\Rightarrow \angle\mathrm{BAD} = \angle\mathrm{CAD} = 50^\circ$

In Right $\triangle\mathrm{ADB}$

$\angle\mathrm{BAD} = 50^\circ, \angle\mathrm{ADB} = 90^\circ$

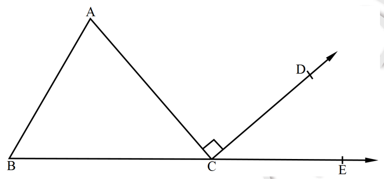
Also sum of all interior angles = 180º

$\Rightarrow \angle\mathrm{BAD} + \angle\mathrm{ADB} + \angle\mathrm{B} = 180^\circ$

$\Rightarrow \angle\mathrm{B} = 180^\circ - 50^\circ - 90^\circ$

$\Rightarrow \angle\mathrm{B} = 40^\circ$

12. In a $\angle\mathrm{ABC}$, it is given that $\angle\mathrm{A} : \angle\mathrm{B} : \angle\mathrm{C} = 3 : 2 : 1$ and $\angle\mathrm{ACD} = 90^\circ$. If BC is produced to E then $\angle\mathrm{ECD} = ?$



(A) 60º (B) 50º (C) 25º (D) 40º

Ans. :

a. 60º

Solution:

Let $\angle\mathrm{A} = (3\mathrm{x})^\circ,\ \angle\mathrm{B} = (2\mathrm{x})^\circ$ and $\angle\mathrm{C} = \mathrm{x}^\circ$

Then,

3x + 2x + x = 180º [Sum of the angles of a triangle]

$\Rightarrow 6x = 180º$

$\Rightarrow x = 30º$

Hence, the angles are

$\angle A = 3 \times 30^\circ = 90^\circ, \ \angle B = 2 \times 30^\circ = 60^\circ$ and $\angle C = 30^\circ$

Side BC of triangle ABC is produced to E.

$\therefore \ \angle ACE = \angle A + \angle B$

$\Rightarrow \angle ACD + \angle ECD = 90^\circ + 60^\circ$

$\Rightarrow 90^\circ + \angle ECD = 150^\circ$

$\Rightarrow \angle ECD = 60^\circ$

13. In a $\triangle ABC$, if $\angle A - \angle B = 42^\circ$ and $\angle B - \angle C = 21^\circ$ then $\angle B = ?$

(A) 63º (B) 32º (C) 95º (D) 53º

Ans. :

d. 53º

Solution:

Let,

$\angle A - \angle B = 42^\circ \ldots \ (i)$ and

$\angle B - \angle C = 21^\circ \ldots \ (ii)$

Adding (i) and (ii),we get

$\angle A - \angle C = 63^\circ \ldots \ (iii)$

$\angle B = \angle A - 42^\circ$ [using (i)]

$\angle C = \angle A - 63^\circ$ [Using (iii)]

$\therefore \angle A + \angle B + \angle C = 180^\circ$ [Sum of the angles of a triangle]

$\Rightarrow \angle A + \angle A - 42^\circ + \angle A - 63^\circ = 180^\circ$

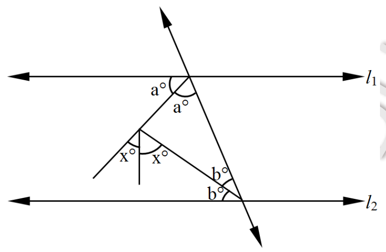
$\Rightarrow 3\angle A - 105^\circ = 180^\circ$

$\Rightarrow 3\angle A = 285^\circ$

$\therefore \angle B = (95 - 42)^\circ$

$\Rightarrow \angle B = 53^\circ$

14. In Figure, if $l_1 || l_2$, the value of x is:

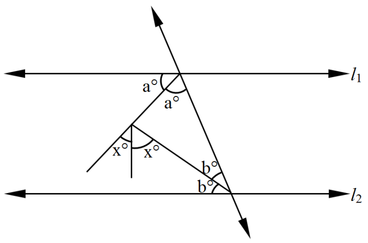


(A) 60 (B) $22\frac{1}{2}$ (C) 45 (D) 30

Ans. :

c. 45

Solution:



$\mathrm{ACS} = 180^\circ - 2\mathrm{b}^\circ$

Also $\angle\mathrm{ACS} = \angle\mathrm{PAC} = 2\mathrm{a}^\circ$ (alternate angles)

⇒ 2aº = 180º − 2bº

⇒ aº + bº = 90º

Now, in $\triangle\mathrm{ABC}$

$\mathrm{a}^\circ + \mathrm{b}^\circ + \angle\mathrm{ABC} = 180^\circ$

$\angle\mathrm{ABC} = 180^\circ - 2\mathrm{x}^\circ$

⇒ aº + bº + 180º − 2xº = 180°

⇒ 2xº = aº + bº = 90º

⇒ xº = 45º

15. In the following, write the correct answer.

Two sides of a triangle are of lengths 5cm and 1.5cm. The length of the third side of the triangle cannot be:

(A) 3.6cm (B) 4.1cm (C) 3.8cm (D) 3.4cm

Ans. :

d. 3.4cm

Solution:

Since sum of any two sides of triangle is always greater than thrid side, so that side of the triangle cannot be 3.4cm becouse then,

1.5cm + 3.4cm = 4.9cm.

16. In $\triangle\mathrm{ABC}$ and $\triangle\mathrm{DEF}$ its is given that $\angle\mathrm{B} = \angle\mathrm{E}$ and $\angle\mathrm{C} = \angle\mathrm{F}$ in order that $\triangle\mathrm{ABC} \cong \triangle\mathrm{DEF}$ we must have,

(A) AC = DE (B) BC = EF (C) AB = DF (D) $\angle\mathrm{A} = \angle\mathrm{D}$

Ans. :

b. BC = EF

Solution:

In $\triangle\mathrm{ABC}$ and $\triangle\mathrm{DEF}$

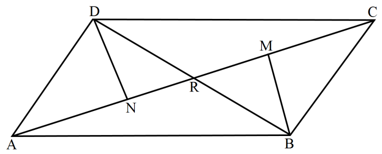
$\angle\mathrm{B} = \angle\mathrm{E}$ and $\angle\mathrm{C} = \angle\mathrm{F}$

For congruence, BC = EF

Therefore by AAS axiom

$\triangle\mathrm{ABC} \cong \triangle\mathrm{DEF}$

17. In quadrilateral ABCD, BM and DN are drawn perpendiculars to AC such that BM = DN. If BR = 8cm. then BD is:



(A) 12cm (B) 2cm (C) 16cm (D) 4cm

Ans. :

c. 16cm

Solution:

In triangles $\triangle \mathrm{DNR}$ and $\triangle \mathrm{BMR}$,

$\angle \mathrm{N} = \angle \mathrm{M} = 90^\circ$

$\angle \mathrm{NRD} = \angle \mathrm{MRB}$ (vertically opposite angles)

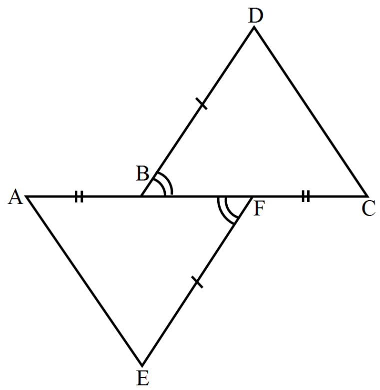
BM = DN (Given)

Therefore, $\triangle \mathrm{DNR}$ and $\triangle \mathrm{MRB}$, are congruent

Therefore, BR = DR = 8cm

BD = 16cm

18. In the adjoining figure, AB = FC, EF = BD and $\angle \mathrm{AFE} = \angle \mathrm{CBD}$. Then the rule by which $\triangle \mathrm{AFE} \cong \triangle \mathrm{CBD}$.



(A) SAS (B) AAS (C) SSS (D) ASA

Ans. :

a. SAS

Solution:

ASA

In $\triangle \mathrm{DBC}$ and $\triangle \mathrm{AEF}$, we have

AB = FC (given) by adding BF on both sides

AF = CB

$\triangle \mathrm{AEF} = \ \triangle \mathrm{CBD}$ (given)

EF = BD (given)

Hence, $\triangle \mathrm{AFE} \cong \triangle \mathrm{CBD}$ by SAS as the corresponding sides and their included angles are equal.

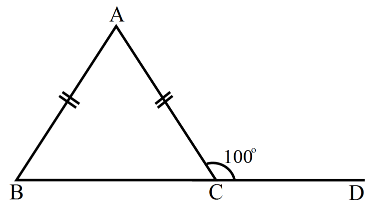
19. In a $\triangle \mathrm{ABC}$, if $\mathrm{AB} = \mathrm{AC}$ and BC is produced to D such that $\angle \mathrm{ACD} = 100^\circ$ then $\angle \mathrm{A} =$

(A) 20° (B) 40° (C) 60° (D) 80°

Ans. :

a. 20°

Solution:



$\mathrm{AB} = \mathrm{AC}$

$\Rightarrow \angle \mathrm{ABC} = \angle \mathrm{ACB}$ (Isoscles $\triangle$ Property)

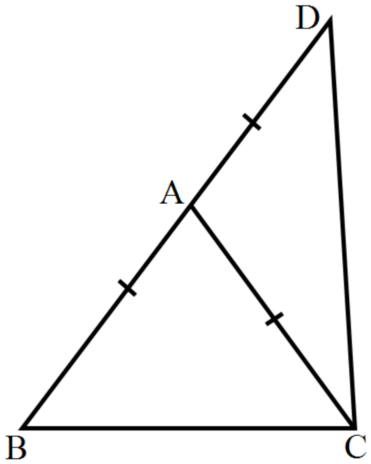
$\angle \mathrm{ACB} = 180^\circ - 100^\circ = 80^\circ$

$\Rightarrow \angle \mathrm{ABC} - = \angle \mathrm{ACB} = 80^\circ$

$\angle \mathrm{A} = 180^\circ - \angle \mathrm{ACB} - \angle \mathrm{ABC} = 180^\circ - 80^\circ - 80^\circ = 20^\circ$

Hence, correct option is (a).

20. In an isosceles, $\triangle \mathrm{ABC}$, AB = AC and side BA is produced to D such that AB = AD. Then the measure of $\angle \mathrm{BCD}$ is:

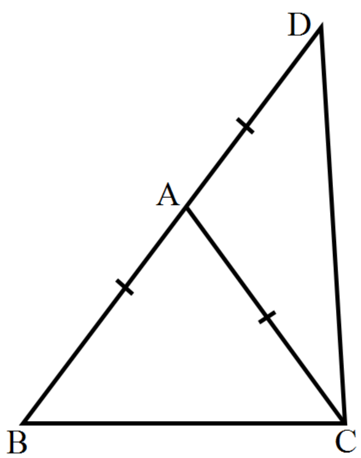


(A) 70º (B) 90º (C) 100º (D) 60º

Ans. :

b. 90º

Solution:



Given in $\triangle \mathrm{ABC}$, AB = AC

$\Rightarrow \angle \mathrm{ABC} = \angle \mathrm{ACB}$ (Since angles opposite to equal sides are equal)

Also given that AD = AB

$\Rightarrow \angle \mathrm{ADC} = \angle \mathrm{ACD}$ (Since angles opposite to equal sides are equal)

$\therefore \angle \mathrm{ABC} = \angle \mathrm{ACB} = \angle \mathrm{ADC} = \angle \mathrm{ACD} = \mathrm{x} (\mathrm{AB} = \mathrm{AC} = \mathrm{AD})$

Also, $\angle BCD = \angle ACB + \angle AC + \angle AD = x + x = 2x$

In $\triangle BCD,\ \angle CBD + \angle BCD + \angle BDC = 180^\circ$

$x + 2x + x = 180^\circ$

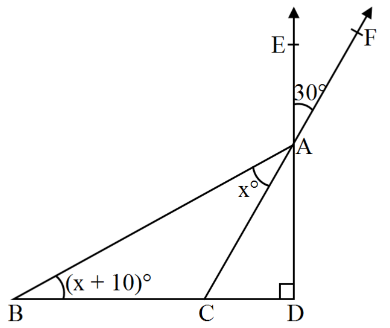
$4x = 180^\circ$

$x = 45^\circ$

$\triangle BCD = 2x$

$= 90^\circ$

21. In the given figure, $EAD \perp BCD$. Ray FAC cuts ray EAD at a point A such that $\angle EAF = 30^\circ$. Also, in $\triangle BAC,\ \angle BAC = x^\circ$ and $\angle ABC = (x + 10)^\circ$. Then, the value of x is:



(A) 30 (B) 20 (C) 35 (D) 25

Ans. :

d. 25

Solution:

In the given figure $\angle CAD = \angle EAF$ (Vertically opposite angels)

$\therefore \angle CAD = 30^\circ$

In $\triangle ABD,$

$\angle ABD + \angle BAD + \angle ADB = 180^\circ$ (Angle sum property)

$\Rightarrow (x + 10)^\circ + (x^\circ + 30^\circ) + 90^\circ = 180^\circ$

$\Rightarrow 2x + 130^\circ = 180^\circ$

$\Rightarrow 2x = 180^\circ - 130^\circ = 50^\circ$

$\Rightarrow x = 25$

Thus, the value of x is 25.

22. If the measure of angles of a triangle are in the ratio of 3 : 4 : 5, what is the measure of the smallest angle of the triangle?

(A) 25º (B) 30º (C) 45º (D) 60º

Ans. :

c. 45º

Solution:

The measures of angles of a triangle are in ratio 3 : 4 : 5.

Let the angles be 3x, 4x, and 5x.

in any triangle, sum of all angles = 180º

⇒ 3x + 4x + 5x = 180º

⇒ 12x = 180º

$\Rightarrow$ x = 15º

So, smallest angle = 3 × 15º = 45º

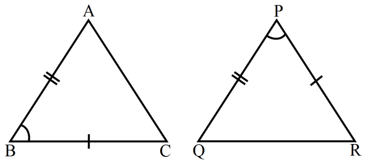
23. In triangles ABC and PQR three equality relation between some parts are as follows:
$\mathrm{AB} = \mathrm{QP}, \angle\mathrm{B} = \angle\mathrm{P}$ and $\mathrm{BC} = \mathrm{PR}$
State which of the congruence conditions applies:

(A) SAS (B) ASA (C) SSS (D) RHS

Ans. :

a. SAS

Solution:



From given conditions, we have

$\mathrm{AB} = \mathrm{PQ}$

$\mathrm{BC} = \mathrm{PR}$

And the angle between these sides are also equal

i.e. $\angle\mathrm{B} = \angle\mathrm{P}$

So SAS property.

Hence, correct option is (a).

24. In an isosceles $\triangle\mathrm{ABC}$, if AB = AC and $\angle\mathrm{A} = 90^\circ$, Find $\angle\mathrm{B}$.

(A) 60° (B) 80° (C) 45° (D) 95°

Ans. :

d. 45º

Solution:

We know that sum of all angle of a triangle is 180°

So, $\angle\mathrm{A} + \angle\mathrm{B} + \angle\mathrm{C} = 180^\circ$

$\angle\mathrm{A} = 90^\circ$

AB = AC

$\angle\mathrm{B} = \angle\mathrm{C}$ (The angle opposite to equal side is also equal)

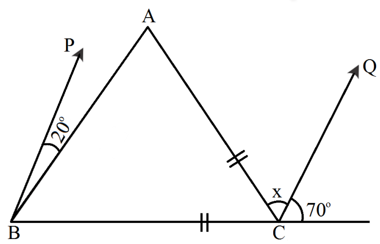
$90^\circ + \angle\mathrm{B} + \angle\mathrm{C} = 180^\circ$

$\angle\mathrm{B} + \angle\mathrm{C} = 180^\circ - 90^\circ$

$\angle\mathrm{B} + \angle\mathrm{C} = 90^\circ (\angle\mathrm{B} = \angle\mathrm{C})$

$2\angle\mathrm{B} = 90^\circ$

25. In Fig. if BP || CQ and AC = BC, then the measure of x is:

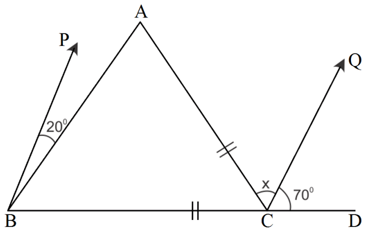


(A) 20º (B) 25º (C) 30º (D) 35º

Ans. :

c. 30º

Solution:



$\angle PBC = \angle QCD$ (Corresponding angles, OP || CQ and BC is transverse)

$\Rightarrow \angle PBC = 70^\circ$

Now, $\angle PBA + \angle ABC + \angle PBC$

$\Rightarrow 20^\circ + \angle ABC = 70^\circ$

$\Rightarrow \angle ABC = 50^\circ$

In $\triangle ABC$,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ \ldots (1)$

Now, $\angle ABC = \angle BAC = 50^\circ$ (isosceles $\triangle$)

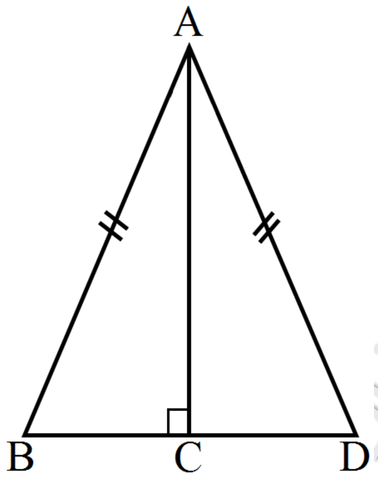
And, $\angle ACB = 180^\circ - (70^\circ + x)$

From (1)

$50^\circ + 50^\circ + 180^\circ - (70^\circ + x) = 180^\circ$

$\Rightarrow x = 30^\circ$

26. In the adjoining figure, AB = AC and $AD \perp BC$. The rule by which $\triangle ABD \cong \triangle ACD$ is:



(A) SAS (B) ASA (C) SSS (D) RHS

Ans. :

d. RHS

Solution:

In $\triangle ABD$ and $\triangle ACD$, we have

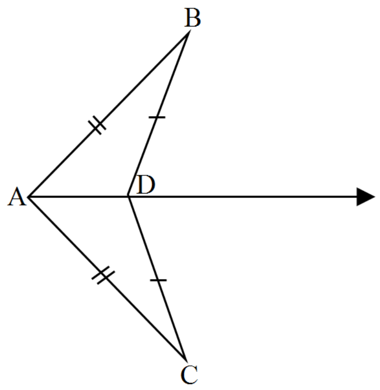
$\angle ADB = \angle ADC$ (Right angles)

AB = AC (Given and hypotenuses)

AD = AD (common in both)

Therefore, $\triangle ABD \cong \triangle ACD$ by RHS.

27. In fig., $\triangle ABD \cong \triangle ACD$, AB = AC, BD = DC name the criteria by which the triangles are congruent:



(A) SSS (B) ASA (C) RHS (D) SAS

Ans. :

a. SSS

Solution:

Given that two sides are equal and third side is common I.e. AD hence all three corresponding sides are equal.

28. In a $\triangle ABC$, if $3\angle A = 4\angle B = 6\angle C$ then A : B : C = ?

(A) 4 : 3 : 2 (B) 6 : 4 : 3 (C) 2 : 3 : 4 (D) 3 : 4 : 6

Ans. :

d. 3 : 4 : 6

Solution:

In the given figure, $\angle ACB + \angle ACD = 180^\circ$ (Linear pair of angles)

∴ 5yº + 7yº = 180º

⇒ 12yº = 180º

⇒ y = 15

In $\triangle ABC$

$\angle A + \angle B + \angle ACB = 180^\circ$ (Angle sum property)

∴ 3yº + xº + 5yº = 180º

⇒ xº + 8yº = 180º

⇒ xº + 8 × 15º = 180º

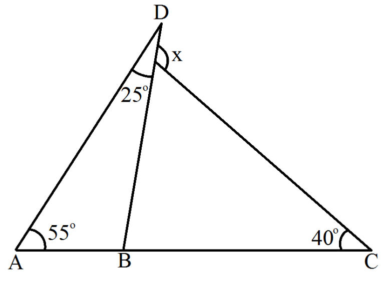
⇒ xº + 120º = 180º

⇒ xº = 180º − 120º = 60º

Thus, the value of x is 60º.

Hence the correct answer is 3 : 4 : 6

29. In Fig the value of x is:

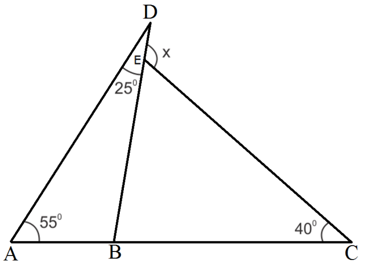


(A) 65º (B) 80º (C) 95º (D) 120º

Ans. :

d. 120º

Solution:



In $\triangle \mathrm{ABD}$

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{D}=180^{\circ}$

$\Rightarrow 55^{\circ}+\angle \mathrm{DBA}+25^{\circ}=180^{\circ}$

$\Rightarrow \angle \mathrm{DBA}=180^{\circ}-55^{\circ}-25^{\circ}$

$=180^{\circ}-80^{\circ}$

$\Rightarrow \angle \mathrm{DBA}=100^{\circ}$

So, $\angle \mathrm{DBC}=180^{\circ}-\angle \mathrm{DBA}$

$=180^{\circ}-100^{\circ}$

$\Rightarrow \angle \mathrm{DBC}=80^{\circ}$

Now, in $\triangle \mathrm{EBC}$

$\angle \mathrm{E}+\angle \mathrm{EBC}+\angle \mathrm{C}=180^{\circ}$

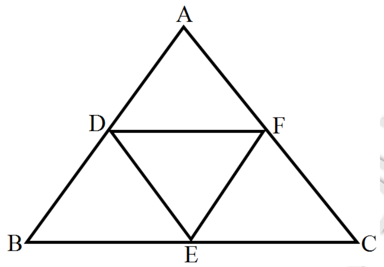
$\Rightarrow \angle \mathrm{E}+80^{\circ}+40^{\circ}=180^{\circ}\ (\angle \mathrm{DBC}=\angle \mathrm{EBC})$

$\Rightarrow \angle \mathrm{E}=180^{\circ}-120^{\circ}=60^{\circ}$

Also, $\mathrm{x}=180^{\circ}-\angle \mathrm{E}=180^{\circ}-60^{\circ}$

$\Rightarrow \mathrm{x}=120^{\circ}$

30. D, E and F are the mid points of sides AB, BC and CA of $\triangle \mathrm{ABC}$. If perimetre of $\triangle \mathrm{ABC}$ is 16cm, then perimetre of $\triangle \mathrm{DEF}$.



(A) None of these (B) 8cm (C) 4cm (D) 32cm

Ans. :

b. 8cm

Solution:

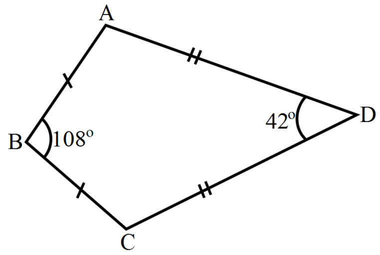
Using relation

perimeter, $\triangle \mathrm{DEF}=\frac{1}{2}$

perimeter, $\triangle \mathrm{ABC}$

$=\frac{1}{2} \times 16=8 \mathrm{~cm}$

31. In figure, ABCD is a quadrilateral in which AB = BC and AD = DC. The measure of $\angle \mathrm{BCD}$ is:



(A) 30º (B) 105º (C) 150º (D) 72º

Ans. :

b. 105º

Solution:

Join AC. We get two isosceles triangles. $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ACD}$.

In $\triangle \mathrm{ABC},\ \angle \mathrm{ABC} = 108^\circ$

$\therefore \angle \mathrm{BAC} = \angle \mathrm{BCA} = \frac{(180^\circ - 108^\circ)}{2} = \frac{72^\circ}{2} = 36^\circ$

In $\triangle \mathrm{ACD},\ \angle \mathrm{ADC} = 42^\circ$

$\therefore \angle \mathrm{DAC} = \angle \mathrm{DCA} = \frac{(180^\circ - 42^\circ)}{2} = \frac{138^\circ}{2} = 69^\circ$

Now, $\angle \mathrm{BCD} = \angle \mathrm{BCA} + \angle \mathrm{DCA} = 36^\circ + 69^\circ = 105^\circ$

32. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is:

(A) 100º (B) 120º (C) 110º (D) 130º

Ans. :

b. 120º

Solution:

Let $\triangle \mathrm{ABC}$ be an isosceles triangle with

vertex angle = $\angle \mathrm{A}$ and base angles = $\angle \mathrm{B}$ and $\angle \mathrm{C}$

Now, $\angle \mathrm{A} = 2(\angle \mathrm{B} + \angle \mathrm{C})$

$\Rightarrow \frac{\angle \mathrm{A}}{2} = \angle \mathrm{B} + \angle \mathrm{C} \ldots .(1)$

Also in $\triangle \mathrm{ABC}$,

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ$

$\Rightarrow \angle \mathrm{A} + (\angle \mathrm{B} + \angle \mathrm{C}) = 180^\circ$

$\Rightarrow \angle \mathrm{A} + \frac{\angle \mathrm{A}}{2} = 180^\circ$ .....[From (1)]

$\Rightarrow \frac{3}{2}\angle \mathrm{A} = 180^\circ$

$\Rightarrow \angle \mathrm{A} = \frac{180^\circ \times 2}{3}$

$\Rightarrow \angle \mathrm{A} = 120^\circ$

Hence, correct option is (b).

33. In $\triangle \mathrm{ABC}$, if $\angle \mathrm{A} = 45^\circ$ and $\angle \mathrm{B} = 70^\circ$, then the shortest and the longest sides of the triangle are ________ .

(A) BC, AB (B) BC, AC (C) AB, BC (D) AB, AC

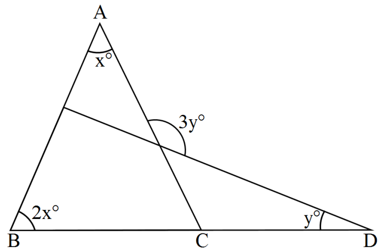
Ans. :

b. BC, AC

Solution:

Smallest angle is A and greatest angle is B and hence sides opposite to these angles are BC and AC and they are shortest and longest respectively.

34. In figure, what is Y in terms of X?



(A) $\frac{3}{2}\mathrm{X}^{\circ}$ (B) $\frac{3}{4}\mathrm{X}^{\circ}$ (C) $\frac{4}{3}\mathrm{X}^{\circ}$ (D) X

Ans. :

a. $\frac{3}{2}\mathrm{X}^{\circ}$

Solution:

From Figure, $\angle\mathrm{DOC} = 180^{\circ} - \angle\mathrm{AOD}$ (Both are Supplementary)

$\Rightarrow\ \angle\mathrm{DOC} = 180^{\circ} - 3\mathrm{y}^{\circ}$

Also, $\angle\mathrm{ACB} = 180^{\circ} - \angle\mathrm{A} - \angle\mathrm{B}$

$\Rightarrow \angle\mathrm{ACB} = 180^{\circ} - \mathrm{x}^{\circ} - 2\mathrm{x}^{\circ} = 180^{\circ} - 3\mathrm{x}^{\circ}$

And $\angle\mathrm{ACD} = 180^{\circ} - \angle\mathrm{ACB}$

$= 180^{\circ} - (180^{\circ} - 3\mathrm{x}^{\circ})$

$\Rightarrow \angle\mathrm{ACD} = 3\mathrm{x}^{\circ}$

Now, in $\triangle\mathrm{OCD}$

$\angle\mathrm{DOC} + \angle\mathrm{OCD} + \angle\mathrm{D} = 180^{\circ}$

$180^{\circ} - 3\mathrm{y}^{\circ} + 3\mathrm{x}^{\circ} + \mathrm{y}^{\circ} = 180^{\circ}\ [\angle\mathrm{OCD} = \angle\mathrm{ACD}]$

$\Rightarrow 2\mathrm{y}^{\circ} = 3\mathrm{x}^{\circ}$

$\Rightarrow\ \mathrm{Y} = \frac{3}{2}\mathrm{X}^{\circ}$

35. The cost of turfing a triangular field at the rate of Rs. 45 per 100m^2 is Rs. 900. If the double the base of the triangle is 5 times its height, then its height is:

(A) 42cm (B) 32cm (C) 44cm (D) 40cm

Ans. :

d. 40cm

Solution:

Cost of turfing a triangilar field at the rate of Rs. 45 per 100 = Rs. 900

$\frac{\mathrm{Arae}\times 45}{100} = 900$

⇒ Area = 2000 sq.cm

According to question,

2 × Base = 5 × Height

$\Rightarrow\ \mathrm{Base} = \frac{\mathrm{Height}\times 5}{2}$

Area of a triangle = 2000 sq.cm

$\Rightarrow \ \frac{1}{2} \times \text{Base} \times \text{Height} = 2000$

$\Rightarrow \ \frac{1}{2} \times \frac{\text{Height} \times 5}{2} \times \text{Height} = 2000$

$\Rightarrow \ (\text{Height})^2 = 1600$

$\Rightarrow \ \text{Height} = 40\text{cm}$

36. If ABC and DEF are two triangles such that $\triangle \mathrm{ABC} \cong \triangle \mathrm{FDE}$ and $\mathrm{AB} = 5\mathrm{m}, \angle \mathrm{B} = 40^\circ$ and $\angle \mathrm{A} = 80^\circ$. Then, which of the following is true?

(A) $\mathrm{DF} = 5\mathrm{cm}, \angle \mathrm{F} = 60^\circ$ (B) $\mathrm{DE} = 5\mathrm{cm}, \angle \mathrm{E} = 60^\circ$ (C) $\mathrm{DF} = 5\mathrm{cm}, \angle \mathrm{E} = 60^\circ$ (D) $\mathrm{DE} = 5\mathrm{cm}, \angle \mathrm{D} = 40^\circ$

Ans. :

c. $\mathrm{DF} = 5\mathrm{cm}, \angle \mathrm{E} = 60^\circ$

Solution:

In $\triangle \mathrm{ABC}$,

$\angle \mathrm{C} = 180^\circ - \angle \mathrm{A} + \angle \mathrm{B} = 180^\circ - 80^\circ - 40^\circ = 60^\circ$

$\triangle \mathrm{ABC} \cong \triangle \mathrm{FDE}$

$\Rightarrow \mathrm{AB} = \mathrm{FD} = 5\mathrm{cm}$

$\Rightarrow \angle \mathrm{B} = \angle \mathrm{D} = 40^\circ$

$\Rightarrow \angle \mathrm{A} = \angle \mathrm{F} = 80^\circ$

$\Rightarrow \angle \mathrm{C} = \angle \mathrm{E} = 60^\circ$

$\Rightarrow \mathrm{DF} = \mathrm{FD} = 5\mathrm{cm}$ and $\angle \mathrm{E} = 60^\circ$

Hence, correct option is (c).

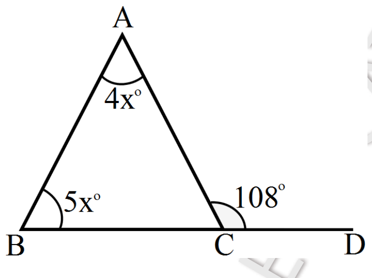
37. An exterior angle of a triangle is 108º and its interior opposite angles are in the ratio 4 : 5 The angles of the triangle are:

(A) 48º, 60º, 72º (B) 50º, 60º, 70º (C) 52º, 56º, 72º (D) 42º, 60º, 76º

Ans. :

a. 48º, 60º, 72º

Solution:



From figure, we have

$\angle \mathrm{A} + \angle \mathrm{B} = \angle \mathrm{ACD}$

$\Rightarrow 4\mathrm{x}^\circ + 5\mathrm{x}^\circ = 108^\circ$

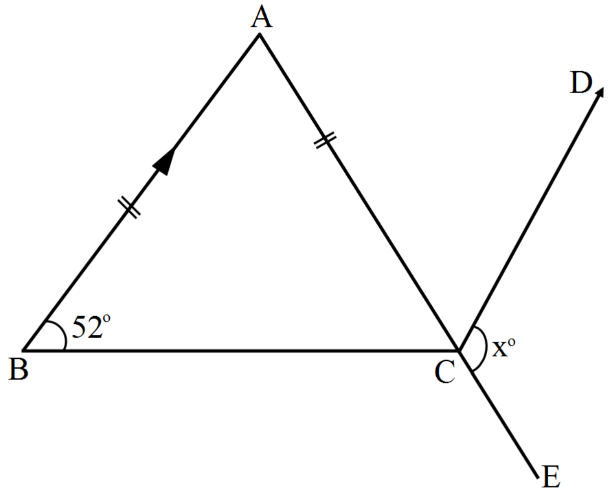
$\Rightarrow 9\mathrm{x}^\circ = 108^\circ$

$\Rightarrow \mathrm{x} = 12^\circ$

So, $\angle \mathrm{A} = 48^\circ, \angle \mathrm{B} = 60^\circ$

$\Rightarrow \angle \mathrm{C} = 180^\circ - 48^\circ - 60^\circ = 72^\circ$

38. In Fig. ABC ids an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x is:

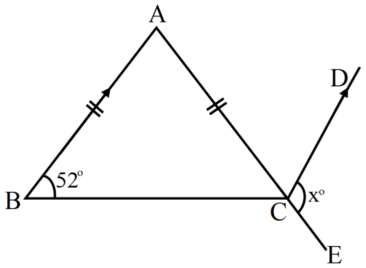


(A) 52º (B) 76º (C) 156º (D) 104º

Ans. :

d. 104º

Solution:



$\triangle\mathrm{ABC}$ is isosceles

$\angle\mathrm{ABC} = \angle\mathrm{ACB} = 52^\circ$

then $\angle\mathrm{BAC} = 180^\circ - 52^\circ - 52^\circ = 76^\circ$

If $\mathrm{AB} \| \mathrm{CD}$, AC is transversal

then $\angle\mathrm{BAC} = \angle\mathrm{ACD}$ (alternate angles)

$\Rightarrow \angle\mathrm{ACD} = 76^\circ$

Now from figure,

$\angle\mathrm{ACD} + \mathrm{x}^\circ = 180^\circ$

$\Rightarrow \mathrm{x}^\circ = 180^\circ - 76^\circ$

$\Rightarrow \mathrm{x}^\circ = 104^\circ$

Hence, correct option is (d).

39. In a $\triangle\mathrm{ABC}$, if $3\angle\mathrm{A} = 4\angle\mathrm{B} = 6\angle\mathrm{C}$ then A : B : C =?

(A) 3 : 4 : 6 (B) 4 : 3 : 2 (C) 2 : 3 : 4 (D) 6 : 4 : 3

Ans. :

b. 4 : 3 : 2

Solution:

Given that $3\angle\mathrm{A} = 4\angle\mathrm{B} = 6\angle\mathrm{C} = \mathrm{k}$.

$\Rightarrow \angle\mathrm{A} = \frac{\mathrm{k}}{3}, \angle\mathrm{B} = \frac{\mathrm{k}}{4}$ and $\angle\mathrm{C} = \frac{\mathrm{k}}{6}$

$\Rightarrow \mathrm{A} : \mathrm{B} : \mathrm{C} = \frac{\mathrm{k}}{3} : \frac{\mathrm{k}}{4} : \frac{\mathrm{k}}{6}$

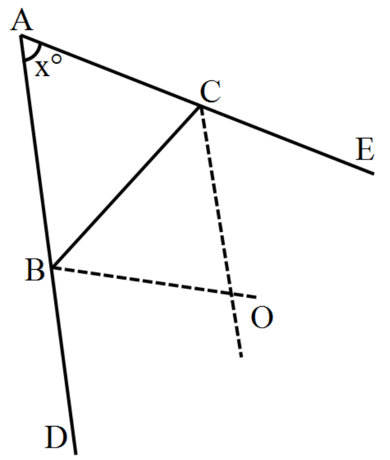
$\Rightarrow \mathrm{A} : \mathrm{B} : \mathrm{C} = \frac{1}{3} : \frac{1}{4} : \frac{1}{3}$

The LCM of 3, 4 and 6 is 12.

Multiply by 12 throughout.

⇒ A : B : C = 4 : 3 : 2

40. The bisector of exterior angles at B and C of $\triangle \mathrm{ABC}$ meet at O. If $\angle \mathrm{A}=\mathrm{x}^{\circ}$, then $\angle \mathrm{BOC}$.



(A) $90^{\circ}-\frac{\mathrm{x}^{\circ}}{2}$ (B) $180^{\circ}+\frac{\mathrm{x}^{\circ}}{2}$ (C) $90^{\circ}+\frac{\mathrm{x}^{\circ}}{2}$ (D) $180^{\circ}-\frac{\mathrm{x}^{\circ}}{2}$

Ans. :

a. $90^{\circ}-\frac{\mathrm{x}^{\circ}}{2}$

Solution:

In $\triangle \mathrm{ABC}$

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}$

$\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}-\mathrm{x}^{\circ} \ldots$ (i)

$\angle \mathrm{CBD}=180^{\circ}-\angle \mathrm{B} \ldots$ (ii)

$\angle \mathrm{ECB}=180^{\circ}-\angle \mathrm{C} \ldots$ (iii)

$\Rightarrow \frac{\angle \mathrm{CBD}}{2}=\angle \mathrm{OBC}=90^{\circ}-\frac{\angle \mathrm{B}}{2} \ldots$ (iv)

$\Rightarrow \frac{\angle \mathrm{ECB}}{2}=\angle \mathrm{OCB}=90^{\circ}-\frac{\angle \mathrm{C}}{2} \ldots$ (v)

Now, in $\triangle \mathrm{BOC}$

$\angle \mathrm{OBC}+\angle \mathrm{OCB}+\angle \mathrm{BOC}=180^{\circ}$

$\angle \mathrm{BOC}=180^{\circ}-(\angle \mathrm{OBC}+\angle \mathrm{OCB})$

From eq. (iv) and (v)

$\angle \mathrm{BOC}=180^{\circ}-\left(90^{\circ}-\frac{\angle \mathrm{B}}{2}+90^{\circ}-\frac{\angle \mathrm{C}}{2}\right)$

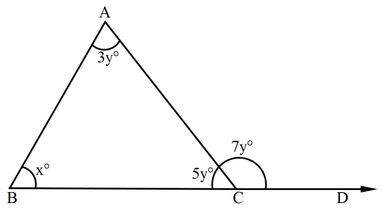
$=180^{\circ}-\left(180^{\circ}-\frac{\angle \mathrm{B}}{2}-\frac{\angle \mathrm{C}}{2}\right)$

$=\frac{\angle \mathrm{B}+\angle \mathrm{C}}{2}$

$=\frac{180^{\circ}+\mathrm{x}^{\circ}}{2}$

$\angle \mathrm{BOC}=90^{\circ}-\frac{\mathrm{x}^{\circ}}{2}$

41. In the given figure, side BC of $\triangle \mathrm{ABC}$ has been produced to a point D. If $\angle \mathrm{A}=3\mathrm{y}^{\circ} \angle \mathrm{B}=\mathrm{x}^{\circ},\ \angle \mathrm{C}=5\mathrm{y}^{\circ}$ and $\angle \mathrm{CBD}=7\mathrm{y}^{\circ}$. Then, the value of x is:



(A) 60 (B) 45 (C) 50 (D) 35

Ans. :

a. 60

Solution:

In the given figure $\angle\mathrm{ACB} + \angle\mathrm{ACD} = 180^\circ$ (Linear pair of angles)

$\therefore 5\mathrm{y}' + 7\mathrm{y}' = 180^\circ$

⇒ 12yº = 180º

⇒ y = 15 ...(i)

In $\triangle\mathrm{ABC},$

$\angle\mathrm{A} + \angle\mathrm{B} + \angle\mathrm{ACB} = 180^\circ$ (angle sum property)

$\therefore 3\mathrm{y}' + \mathrm{x}' + 5\mathrm{y}' = 180^\circ$

⇒ xº + 8yº = 180º

⇒ x^2 + 8 × 15º = 180º [using (1)]

⇒ x′ + 120º = 180º

⇒ xº = 180º − 120º = 60º

Thus, the value of x is 60.

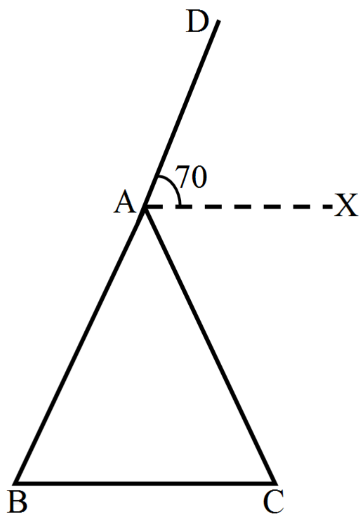
42. In $\triangle\mathrm{ABC}, \angle\mathrm{B} = \angle\mathrm{C}$ and ray AX bisects the exterior angle $\angle\mathrm{DAC}$. If $\angle\mathrm{DAX} = 70^\circ$ then $\angle\mathrm{ACB} =$

(A) 35º (B) 90º (C) 70º (D) 55º

Ans. :

c. 70º

Solution:



AX is bisector of $\angle\mathrm{DAC}$.

$\Rightarrow \angle\mathrm{DAX} = \angle\mathrm{XAC} = 70^\circ$

$\Rightarrow \angle\mathrm{DAC} = 2 \times 70^\circ = 140^\circ$

Now, $\angle A = 180^\circ - \angle DAC = 180^\circ - 140^\circ = 40^\circ$

In $\triangle ABC,$

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 40^\circ + \angle B + \angle C = 180^\circ$

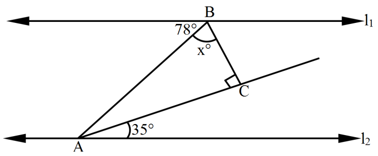
$\Rightarrow 40^\circ + \angle C + \angle C = 180^\circ \ldots (\angle B = \angle C)$

$\Rightarrow 2\angle C = 140^\circ$

$\Rightarrow \angle C = 70^\circ$

$\text{i.e}\angle ACB = 70^\circ$

43. In figure, for which value of x is $11||12?$



(A) 45 (B) 47 (C) 43 (D) 37

Ans. :

b. 47

Solution:

Let if $11||12$ and AB is transverse to it

Then,

$\angle PBA$ should be equal to $\angle BAS$ (Alternate angles)

So if $11||12$, then $\angle BAS = 70^\circ$

$\Rightarrow \angle BAC = 78^\circ - 35^\circ = 43^\circ \ \ldots \ (i)$

Now, in $\angle ABC$

$x^\circ + \angle C + \angle BAC = 180^\circ$

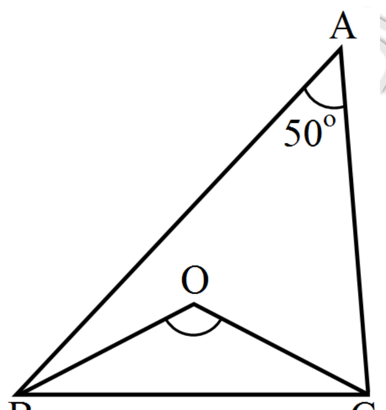
$\Rightarrow x^\circ + 90^\circ + 43^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 90^\circ - 43^\circ = 47^\circ$

$\Rightarrow x^\circ = 47^\circ$

So if $x^\circ = 47^\circ$ then $11||12$

44. In the given figure, BO and CO are bisectors of $\angle B$ and $\angle C$ respectively. If $\angle A = 50^\circ$ then $\angle BOC = ?$



(A) 130º (B) 100º (C) 115º (D) 120º

Ans. :

c. 115º

Solution:

$\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property)

$\Rightarrow 50^\circ + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle B + \angle C = 130^\circ$

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C = 65^\circ$

In $\triangle OBC,$

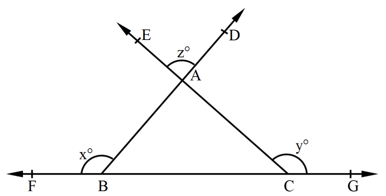
$\angle OBC + \angle OCB + \angle BOC = 180^\circ$ (Angle sum property)

$\Rightarrow \frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BOC = 180^\circ$

$\Rightarrow 65^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 115^\circ$

45. In the given figure, two rays BD and CE intersect at a point A. The side BC of $\triangle ABC$ have been produced on both sides to points F and G respectively. If $\angle ABF = x^\circ,\ \angle ACG = y^\circ$ and $\angle DAE = z^\circ$ then $z = ?$



(A) x + y + 180 (B) 180 - (x + y) (C) x + y - 180 (D) x + y + 360º

Ans. :

c. x + y - 180

Solution:

In the given figure, $\angle ABF + \angle ABC = 180^\circ$ (Linear pair of angles)

$\therefore x^\circ + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - x^\circ \ldots \text{ (i)}$

Also, $\angle ACG + \angle ACB = 180^\circ$ (Linear pair of angles)

$\therefore y' + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - y' \ldots \text{ (ii)}$

Also, $\angle BAC = \angle DAE = z^\circ \ \ldots. \ \text{(iii)}$ (Vertically opposite angles)

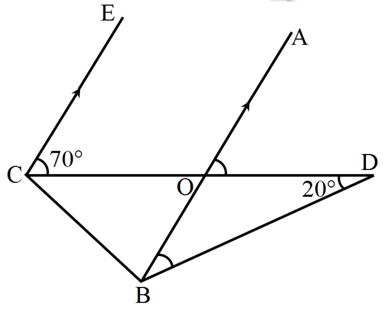
In $\triangle ABC$

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$ (Angle sum property)

$\therefore z^\circ + 180 - x^\circ + 180^\circ - y^\circ = 180^\circ$ [Using (1), (2) and (3)]

$\Rightarrow z = x + y – 180$

46. In Figure, if $EC \parallel AB, \angle ECD = 70^\circ$ and $\angle BDO = 20^\circ$, then $\angle OBD$ is:



(A) 60°   (B) 70°   (C) 20°   (D) 50°

Ans. :

d. 50°

Solution:

$\mathrm{EC} \| \mathrm{AB}$ and CD is transverse to it.

Now $\angle\mathrm{ECD} = \angle\mathrm{AOD} = 70^\circ$ (Corresponding angles)

In $\angle\mathrm{OBD}$

$\angle\mathrm{OBD} + \angle\mathrm{BOD} + \angle\mathrm{ODB} = 180^\circ$

$\angle\mathrm{BOD} = 180^\circ - \angle\mathrm{AOD} = 180^\circ - 70^\circ = 110^\circ$

$\angle\mathrm{ODB} = 20^\circ$ (Given)

So, $\angle\mathrm{OBD} = 180^\circ - \angle\mathrm{BOD} - \angle\mathrm{ODB}$

$= 180^\circ - 110^\circ - 20^\circ$

$= 50^\circ$

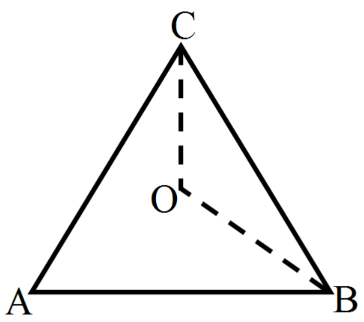
47. In a $\triangle\mathrm{ABC}$, if $\angle\mathrm{A} = 60^\circ, \angle\mathrm{B} = 80^\circ$ and the bisectors of $\angle\mathrm{B}$ and $\angle\mathrm{C}$ meet at O, then $\angle\mathrm{BOC} =$

(A) 60º   (B) 120º   (C) 150º   (D) 30º

Ans. :

b. 120º

Solution:



O is point where bisectors of $\angle\mathrm{C}$ $\&\angle\mathrm{B}$ meets.

$\angle\mathrm{A} + \angle\mathrm{B} + \angle\mathrm{C} = 180^\circ$

$60^\circ + 80^\circ + \angle\mathrm{C} = 108^\circ$

$\angle\mathrm{C} = 40^\circ$

$\frac{\angle\mathrm{C}}{2} = 20^\circ$

$\frac{\angle\mathrm{C}}{2} = 20^\circ = \angle\mathrm{BCO}\ldots(1)$

$\frac{\angle\mathrm{B}}{2} = \frac{80^\circ}{2} = 40^\circ = \angle\mathrm{OBC}\ldots(2)$

In $\triangle\mathrm{BOC}$

$\angle\mathrm{BCO} + \angle\mathrm{OBC} + \angle\mathrm{BOC} = 180^\circ$

from (1) and (2)

$20^\circ + 40^\circ + \angle\mathrm{BOC} = 180^\circ$

$\Rightarrow \angle\mathrm{BOC} = 180^\circ - 60^\circ = 120^\circ$

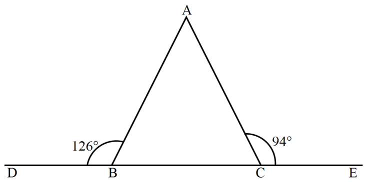
48. The base BC of triangle ABC is produced both ways and the measure of exterior angles formed are 94º and 126º. Then, $\angle\mathrm{BAC} =$

(A) 54º   (B) 94º   (C) 44º   (D) 40º

Ans. :

d. 40º

Solution:



$\angle ABC = 180^\circ - 126^\circ = 54^\circ$

$\angle ACB = 180^\circ - 94^\circ = 86^\circ$

Now, in $\triangle ABC$

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - \angle ABC - \angle ACB$

$= 180^\circ - 54^\circ - 86^\circ$

$\Rightarrow \angle BAC = 40^\circ$

49. In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then, $\angle A = ?$

(A) 50° (B) 40° (C) 100° (D) 80°

Ans. :

a. 50°

Solution:

In $\triangle ABC$,

$BC = AB$

$\Rightarrow \angle A = \angle C$ (angles opposite to equal sides are equal)

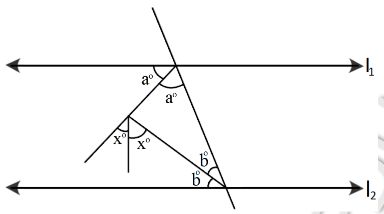
Now, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 80^\circ + \angle A = 180^\circ$

$\Rightarrow 2\angle A + 100^\circ$

$\Rightarrow \angle A = 50^\circ$

50. In Fig. if $l_1 \parallel l_2$, the value of x is:

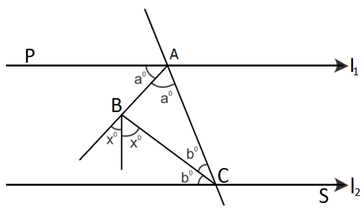


(A) $22\frac{1}{2}$ (B) 30 (C) 45 (D) 60

Ans. :

c. 45

Solution:



From figure,

$\angle ACS = 180^\circ - 2b^\circ$

also $\angle ACS = \angle PAC = 2a^\circ$ (alternate angles)

$\Rightarrow 2a^\circ = 180^\circ - 2b^\circ$

$\Rightarrow a^\circ + b^\circ = 90^\circ$

Now, in $\triangle ABC$

$a^\circ + b^\circ + \angle ABC = 180^\circ$

$\angle ABC = 180^\circ - 2x^\circ$

$\Rightarrow a^\circ + b^\circ + 180^\circ - 2x^\circ = 180^\circ$

$\Rightarrow 2x^\circ = a^\circ + b^\circ = 90^\circ$

$\Rightarrow x^\circ = 45^\circ$

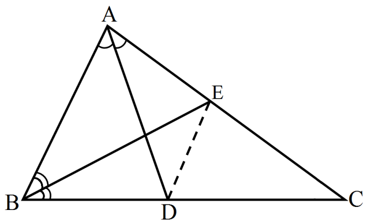
51. In Fig. ABC is a triangle in which $\angle B = 2\angle C.$ D is a point on side BC such that AD bisects $\angle BAC$ and $AB = CD.$ Be is the bisector of $\angle B.$ The measure of $\angle BAC$ is:

(A) 72º (B) 95º (C) 73º (D) 74º

Ans. :

a. 72º

Solution:



$\angle ABE = \angle EBC$ (EBC is bisector of $\angle B$)

and $\angle C = \frac{\angle B}{2}$

$\Rightarrow \angle EBC = \angle ECB$

So $\triangle EBC$ is isosceles triangle.

$\Rightarrow EB = EC \ldots . (1)$

Now Consider $\triangle ABE$ and $\triangle DCE$

$AB = DC$ (Given)

$BE = CE$ [From (1)]

$\angle ABE = \angle DCE$ (From above data)

So $\triangle ABE \cong \triangle DCE$ by SAS property

$\Rightarrow AE = DE$

$\angle BAE = \angle CDE = \angle A$

Now consider $\triangle AED,$

$AE = DE$ (above proved)

$\Rightarrow \triangle AED$ is isosceles triangle

$\Rightarrow \angle EAD = \angle EDA = \frac{\angle A}{2}$ (AD is Bisector of $\angle A$) $\ldots . (2)$

Now, consider $\triangle ABC,$

$\angle A + \angle B + C = 180^\circ$

$\Rightarrow \angle A + 2\angle C + \angle C = 180^\circ (\angle B = 2\angle C)$

$\Rightarrow \angle A + 3\angle C = 180^\circ \ldots\ldots (3)$

Consider $\triangle ADE$,

$\Rightarrow \frac{\angle A}{2} + \angle ADC + \angle C = 180^\circ$

$\Rightarrow \frac{\angle A}{2} + (\angle EDA + \angle CDE) + \angle C = 180^\circ$

$\Rightarrow \frac{\angle A}{2} + \frac{\angle A}{2} + \angle A + \angle C = 180^\circ$

$\Rightarrow \angle A + \angle A + \angle C = 180^\circ$

$\Rightarrow 2\angle A + \angle C = 180^\circ \ldots\ldots (4)$

Right hand side of equations (3) and (4) are equal, hence Left hand side.

$\Rightarrow \angle A + 3\angle C = 2\angle A + \angle C$

$\Rightarrow \angle A = 2\angle C$

Substituting in equation (3),

$2\angle C + 3\angle C = 180^\circ$

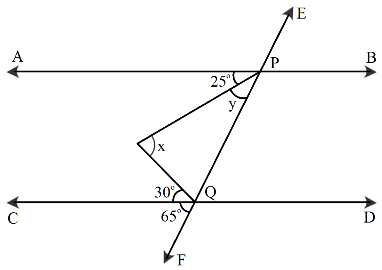
$\Rightarrow 5\angle C = 180^\circ$

$\Rightarrow \angle C = 36^\circ$

$\Rightarrow \angle A = 2 \times 36^\circ = 72^\circ$

Hence, correct option is (a).

52. In Fig. AB and CD are parallel lines and transversal EF intersect them at P and Q respectively. If $\angle APR = 25^\circ, \angle RQC = 30^\circ$ and $\angle CQF = 65^\circ$, then:

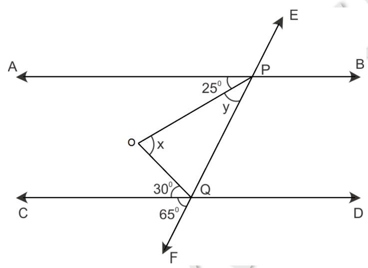


(A) x = 55º, y = 40º (B) x = 50º, y = 45º (C) x = 60º, y = 35º (D) x = 35º, y = 60º

Ans. :

a. x = 55º, y = 40º

Solution:



$\angle OQP = 180^\circ - \angle OQF$

$= 180^\circ - (30^\circ + 65^\circ)$

$\Rightarrow \angle OQP = 85^\circ \ldots (1)$

$\angle APQ = \angle CQF$ (Corresponding angles)

$\Rightarrow 25^\circ + y^\circ = 65^\circ$

$\Rightarrow y^\circ = 65^\circ - 25^\circ$

$\Rightarrow y^\circ = 40^\circ$

Now in $\triangle OPQ$

$\angle O + \angle OPQ + \angle PQO = 180^\circ$

$\Rightarrow x^\circ + 40^\circ + 85^\circ = 180^\circ$

$x^\circ = 180^\circ - 85^\circ - 40^\circ = 55^\circ$

$\Rightarrow x^\circ = 55^\circ, y = 40^\circ$

53. If $\triangle PQR \equiv \triangle EFD$, then $\angle E =$

(A) $\angle Q$ (B) $\angle R$ (C) None of these (D) $\angle P$

Ans. :

d. $\angle P$

Solution:

Since, by corresponding part of congruent $\angle E$ of $\triangle EFD$ is equal to the $\angle P$ of $\triangle PQR$.

54. In the following, write the correct answer.

In $\triangle PQR$ if $\angle R = \angle P$ and QR = 4cm and PR = 5cm. Then, the length of PQ is:

(A) 4cm (B) 5cm (C) 2cm (D) 2.5cm

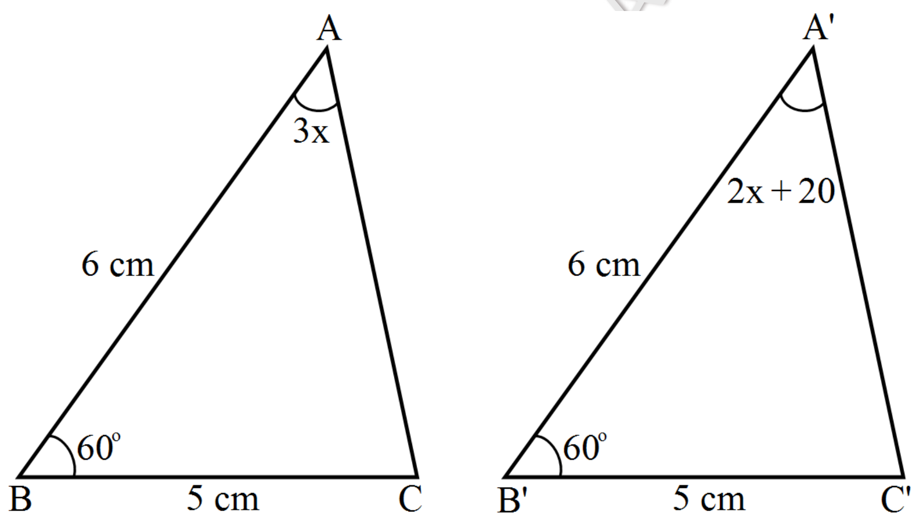
Ans. :

a. 4cm

Solution:

In $\triangle PQR$ if $\angle R = \angle P$  NOw, QR = 4cm, Therefore, PQ = 4cm.

55. In Fig. The measure of $\angle B'A'C'$ is:

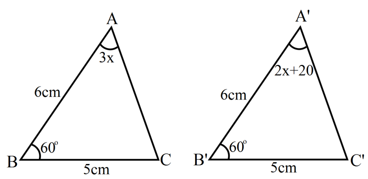


(A) 50º (B) 60º (C) 70º (D) 80°

Ans. :

b. 60º

Solution:



In $\triangle\mathrm{ABC}$ and $\triangle\mathrm{A'B'C}$,

$\mathrm{AB} = \mathrm{A'B'}$

$\mathrm{BC} = \mathrm{B'C'}$

$\angle\mathrm{ABC} = \angle\mathrm{A'B'C'}$

So $\triangle\mathrm{ABC} \cong \triangle\mathrm{A'B'C'}$ by SAS creterion

$\Rightarrow \angle\mathrm{BAC} = \angle\mathrm{B'A'C'}$

$\Rightarrow 3\mathrm{x} = 2\mathrm{x} + 20$

$\mathrm{x} = 20^\circ$

$2\mathrm{x} + 20 = 2 \times 20 + 60^\circ = \angle\mathrm{B'A'C'}$

Hence, correct option is (b).

56. The area of a right angled triangle is 20m^2 and one of the sides containing the right triangle is 4cm. Then the altitude on the hypotenuse is:

(A) $8\mathrm{cm}$ (B) $10\mathrm{cm}$ (C) $\frac{20}{\sqrt{29}}\mathrm{cm}$ (D) $\frac{10}{\sqrt{41}}\mathrm{cm}$

Ans. :

c. $\frac{20}{\sqrt{29}}\mathrm{cm}$

Solution:

Area of right angle triangle = 20 sq.m

⇒ 12 × Base × Height = 20

⇒ 12 × Base × 4 = 20

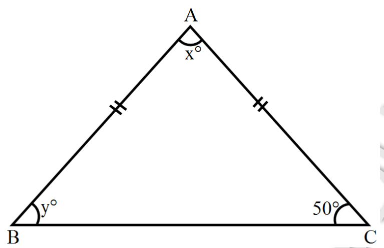
⇒ Base = 10cm

Then, Hypotenuse $= \sqrt{10^2 + 4^2} = 2\sqrt{29}\,\mathrm{m}$

If the altitude drawn to the hypotenuse of a right angle triangle, then the lenght of required altitude

$= \frac{10\times4}{2\sqrt{29}} = \frac{20}{\sqrt{29}}\mathrm{cm}$

57. In the adjoining fig. AB = AC. If $\angle\mathrm{C} = 50^\circ$, then the value of x and y are:



(A) x = 50º and y = 80º (B) x = 60º and y = 70º (C) x = 70º and y = 60º (D) x = 80º and y = 50º

Ans. :

d. x = 80º and y = 50º

Solution:

In triangle ABC, AB = AC, hence their opposite angles will be equal.

$\Rightarrow \angle\mathrm{B} = \angle\mathrm{C} = 50^\circ$

⇒ y = 50$^\mathrm{o}$

Now, by angle sum property,

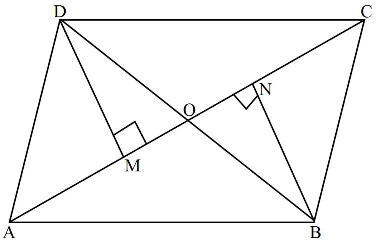
$\angle A + \angle B + \angle C = 180^\circ$

or, x + 50° + 50° = 180°

or, x + 100° = 180°

⇒ x = 80°

58. In the adjoining figure, ABCD is a quadrilateral in which BN and DM are drawn perpendiculars to AC such that BN = DM. If OB = 4cm. then BD is:



(A) 6cm (B) 8cm (C) 10cm (D) 12cm

Ans. :

b. 8cm

Solution:

In Triangle DMO and triangle BNO,

BN = DM and $\angle DMO = \angle BNO$ (90°)

$\angle DMO = \angle BNO$

Therefore, Triangle DMO and triangle BNO are congruent by AAS criteria

Therefore, OB = OD (by CPCT)

So OD = 4cm, BD = OD + OB = 4 + 4 = 8cm

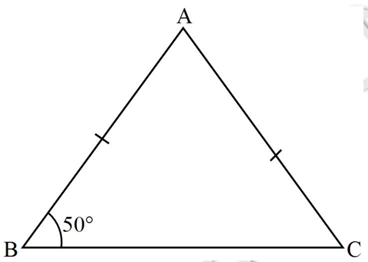
59. In $\triangle ABC$, $AB{=}AC$ and $\angle B = 50^\circ$. Then $\angle A = ?$.

(A) 40 (B) 50 (C) 130 (D) 80

Ans. :

d. 80

Solution:



$AB = AC$ and $\angle B = 50^\circ$

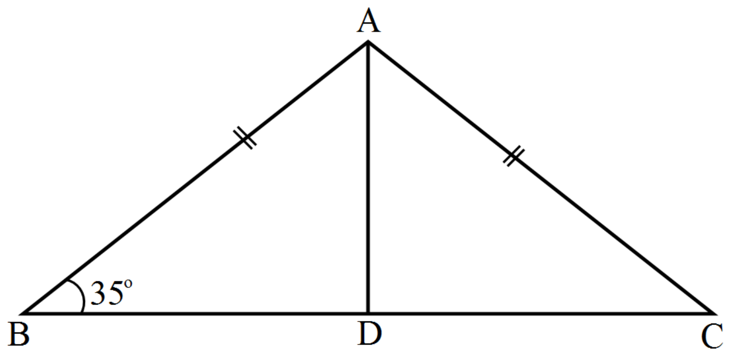
therefore, $\angle C = 50^\circ$ also [angles opposite to equal sides are equal]

$\angle A + \angle B + \angle C = 180^\circ$ [by angle sum property of a triangle]

$\Rightarrow \angle A + 50^\circ + 50^\circ = 180^\circ$

$\Rightarrow \angle A = 80^\circ$

60. ABC is an isosceles triangle such that AB = AC and AD is the median to base BC. Then, $\angle BAD =$

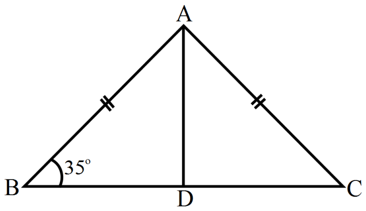


(A) 55° (B) 70° (C) 35° (D) 110°

Ans. :

a. 55°

Solution:



If AD is the median, then D is the mid-point of BC.

$\mathrm{BD} = \mathrm{DC}$

So consider $\triangle \mathrm{ADB}$ and $\triangle \mathrm{ADC}$

$\mathrm{AD} = \mathrm{AD}$ (common)

$\mathrm{DB} = \mathrm{DC}$

$\mathrm{BA} = \mathrm{CA}$

So by SSS, $\triangle \mathrm{ADB} \cong \triangle \mathrm{ADC}$

Now $\angle \mathrm{B} = \angle \mathrm{C} = 35^\circ$

$\Rightarrow \angle \mathrm{BAD} = \angle \mathrm{DAC}$

So in $\triangle \mathrm{ABC}$,

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ$

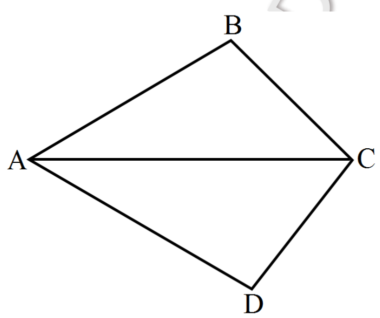
$\Rightarrow 2\angle \mathrm{BAD} + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow 2\angle \mathrm{BAD} = 110^\circ$

$\Rightarrow \angle \mathrm{BAD} = 55^\circ$

Hence, correct option is (a).

61. In the adjoining figure, $\triangle \mathrm{ABC} \cong \triangle \mathrm{ADC}$. If $\angle \mathrm{BAC} = 30^\circ$ and $\angle \mathrm{ABC} = 100^\circ$ then $\angle \mathrm{ACD}$ is equal to:



(A) 80º (B) 60º (C) 30º (D) 50º

Ans. :

d. 50º

Solution:

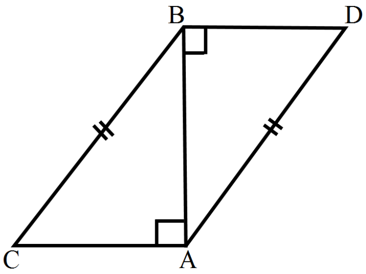
In triangle ABC, $\angle \mathrm{BAC} = 30^\circ$ and $\angle \mathrm{ABC} = 100^\circ$ (Given)

$\angle \mathrm{BAC} + \angle \mathrm{ABC} + \angle \mathrm{BCA} = 180^\circ$

$\angle \mathrm{BAC} = 50^\circ$

Also $\angle \mathrm{ACD} = 50^\circ$ (Since, $\triangle \mathrm{ABC} \cong \triangle \mathrm{ADC}$)

62. In the adjoining figure, BC = AD, $\mathrm{CA} \perp \mathrm{AB}$ and $\mathrm{BD} \perp \mathrm{AB}$. The rule by which $\triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}$ is:



(A) ASA (B) RHS (C) SSS (D) SAS

Ans. :

b. RHS

Solution:

In $\triangle \mathrm{ABC}$ and $\angle \mathrm{BAG} = \angle \mathrm{ABD}$

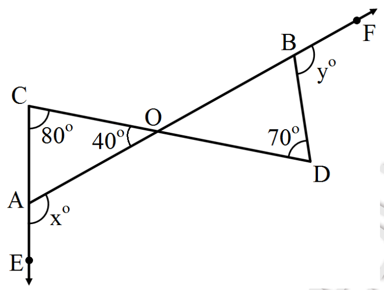
BAD, we have (Right angles)

BC = AD (Hypotenuses and Given)

AB = AB (conunon in both)

Hence, $\triangle \mathrm{ABC} \cong \triangle \mathrm{BAD}$ by RHS criterion.

63. In the given figure, lines AB and CD intersect at a point O. The sides CA and OB have been produced to E and repectively such that $\angle \mathrm{OAE} = \mathrm{x}^\circ$ and $\angle \mathrm{DBF} = \mathrm{y}^\circ$.



If $\angle \mathrm{OCA} = 80^\circ, \angle \mathrm{COA} = 40^\circ$ and $\angle \mathrm{BDO} = 70^\circ$ then $\mathrm{x}^\circ + \mathrm{y}^\circ = ?$

(A) 190º (B) 230º (C) 210º (D) 270º

Ans. :

b. 230º

Solution:

In $\triangle \mathrm{OAC}$, by angle sum property

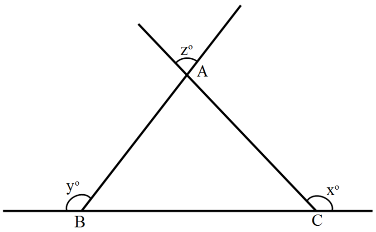
$\angle \mathrm{OCA} + \angle \mathrm{COA} + \angle \mathrm{CAO} = 180^\circ$

$\Rightarrow 80^\circ + 40^\circ + \angle \mathrm{CAO} = 180^\circ$

$\Rightarrow \angle \mathrm{CAO} = 60^\circ$

$\angle CAO + \angle OAE = 180^\circ$ (linear pair)
$\Rightarrow 60^\circ + x = 180^\circ$
$\Rightarrow x = 120^\circ$
$\angle COA = \angle BOD$ (vertically opposite angles)
$\Rightarrow \angle BOD = 40^\circ$
In $\triangle OBD,$ by angle sum property
$\angle OBD + \angle BOD + \angle ODB = 180^\circ$
$\Rightarrow \angle OBD + 40^\circ + 70^\circ = 180^\circ$
$\Rightarrow \angle OBD = 70^\circ$
$\angle OBD + \angle DBF = 180^\circ$ (linear pair)
$\Rightarrow 70^\circ + y = 180^\circ$
$\Rightarrow y = 110^\circ$
$\therefore x + y = 120^\circ + 110^\circ = 230^\circ$

64. In Fig. what is z in terms of x and y?

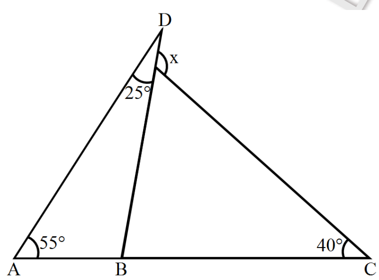


(A) x + y + 180º (B) x + y - 180º (C) 180º - (x + y) (D) x + y + 360º

Ans. :
b. x + y - 180º

Solution:
From figure
$\angle A = z^\circ$
$\angle ACB = 180^\circ - x^\circ$
$\angle ABC = 180^\circ - y^\circ$
Now, in $\triangle ABC$
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow z^\circ + 180^\circ - y^\circ + 180^\circ - x^\circ = 180^\circ$
$\Rightarrow z^\circ = x^\circ + y^\circ - 180^\circ$

65. In figure, the value of x is:



(A) 95º (B) 65º (C) 120º (D) 80º

Ans. :
c. 120º

Solution:

In $\triangle \mathrm{ABD}$

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{D}=180^{\circ}$

$\Rightarrow 55^{\circ}+\angle \mathrm{DBA}+25^{\circ}=180^{\circ}$

$\Rightarrow \angle \mathrm{DBA}=180^{\circ}-55^{\circ}-25^{\circ}$

$=180^{\circ}-80^{\circ}$

$\Rightarrow \angle \mathrm{DBA}=100^{\circ}$

So, $\angle \mathrm{DBC}=180^{\circ}-\angle \mathrm{DBA}$

$=180^{\circ}-100^{\circ}$

$\angle \mathrm{DBC}=80^{\circ}$

Now, $\triangle \mathrm{EBC}$

$\angle \mathrm{A}+\angle \mathrm{EBC}+\angle \mathrm{C}=180^{\circ}$

$\Rightarrow \angle \mathrm{E}+80^{\circ}+40^{\circ}=180^{\circ}(\angle \mathrm{DBC}=\angle \mathrm{EBC})$

$\Rightarrow \angle \mathrm{E}=180^{\circ}-120^{\circ}=60^{\circ}$

Also, $\mathrm{x}=180^{\circ}-\angle \mathrm{E}=180^{\circ}-60^{\circ}$

$\Rightarrow \mathrm{x}=120^{\circ}$

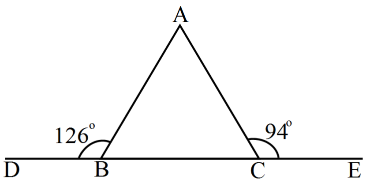
66. The base BC of triangle ABC is produced both ways and the measure of exterior angles formed are 94º and 126º. Then, $\angle \mathrm{BAC}=$

(A) 94º (B) 54º (C) 40º (D) 44º

Ans. :

c. 40º

Solution:



$\angle \mathrm{ABC}=180^{\circ}-126^{\circ}=54^{\circ}$

$\angle \mathrm{ACB}=180^{\circ}-94^{\circ}=86^{\circ}$

Now, in $\triangle \mathrm{ABC}$

$\angle \mathrm{BAC}+\angle \mathrm{ABC}+\angle \mathrm{ACB}=180^{\circ}$

$\Rightarrow \angle \mathrm{BAC}=180^{\circ}-\angle \mathrm{ABC}-\angle \mathrm{ACB}$

$=180^{\circ}-54^{\circ}-86^{\circ}$

$\Rightarrow \mathrm{BAC}=40^{\circ}$

67. Side BC of a triangle ABC has been produced to a point D such that $\angle \mathrm{ACD}=120^{\circ}$ If $\angle \mathrm{B}=12 \angle \mathrm{A}$, then $\angle \mathrm{A}$ is equal to:

(A) 80º (B) 90º (C) 75º (D) 60º

Ans. :

a. 80º

Solution:

$\angle \mathrm{B}=\frac{1}{2} \angle \mathrm{A}$

$\angle \mathrm{ACD}$ is an exterior angle

$\Rightarrow \angle A + \angle B = \angle ACD$

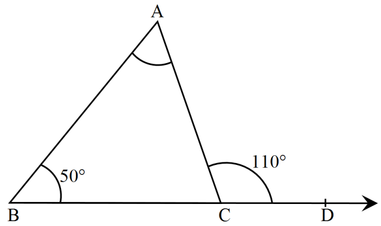
$\Rightarrow \angle A + \frac{1}{2}\angle A = 120^\circ$

$\Rightarrow \frac{3\angle A}{2} = 120^\circ$

$\Rightarrow 3\angle A = 240^\circ$

$\Rightarrow \angle A = 80^\circ$

68. In a $\triangle ABC$, side BC is produced to D. If $ABC = 50^\circ$ and $\angle ACD = 110^\circ$ then $\angle A = ?$



(A) 160° (B) 60° (C) 30° (D) 80°

Ans. :

b. 60°

Solution:

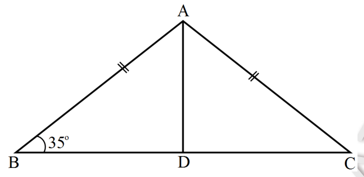
$\therefore \angle A + \angle B = \angle ACD$

$\Rightarrow \angle A + 50^\circ = 110^\circ$

$\Rightarrow \angle A = 60^\circ$

69. ABC is an isosceles triangle such that AB = AC and AD is the median to base BC. Then, $\angle BAD =$

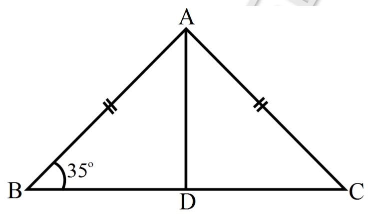
a. 55°
b. 70°
c. 35°
d. 110°



Ans. :

a. 55°

Solution:



If AD is the median, then D is the mid-point of BC.

$BD = DC$

So consider $\triangle ADB$ and $\triangle ADC$

$AD = AD$ (common)

$DB = DC$

$BA = CA$

So by SSS, $\triangle ADB \cong \triangle ADC$

Now $\angle B = \angle C = 35^\circ$

$\Rightarrow \angle BAD = \angle DAC$

So in $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 2\angle BAD + 35^\circ + 35^\circ = 180^\circ$

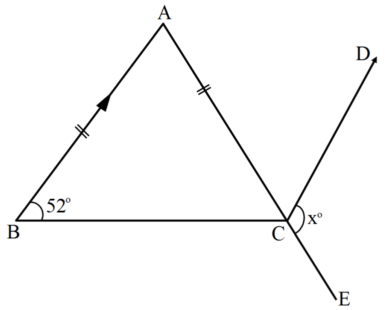
$\Rightarrow 2\angle BAD = 110^\circ$

$\Rightarrow \angle BAD = 55^\circ$

Hence, correct option is (a).

70. In Fig. ABC ids an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x is:

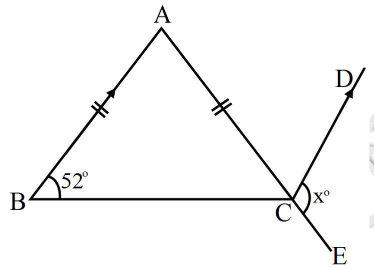
a. 52º
b. 76º
c. 156º
d. 104º



Ans. :

d. 104º

Solution:



$\triangle ABC$ is isosceles

$\angle ABC = \angle ACB = 52^\circ$

then $\angle BAC = 180^\circ - 52^\circ - 52^\circ = 76^\circ$

If $AB \| CD$, AC is transversal

then $\angle BAC = \angle ACD$ (alternate angles)

$\Rightarrow \angle ACD = 76^\circ$

Now from figure,

$\angle ACD + x^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 76^\circ$

$\Rightarrow x^\circ = 104^\circ$

Hence, correct option is (d).

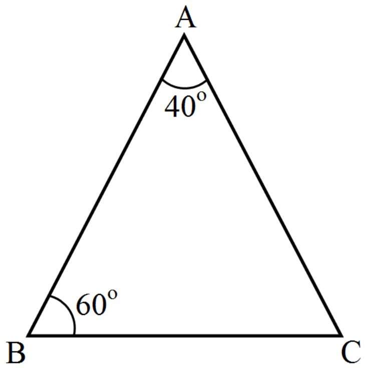
71. In $\triangle ABC$, $\angle A = 40^\circ$ and $\angle B = 60^\circ$. Then the longest side of $\triangle ABC$ is:

a. BC
b. AC
c. AB
d. cannot be determined

Ans. :

c. AB

Solution:



In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^\circ$ ...(Using Angle Sum Property)

$\Rightarrow 40^\circ + 60^\circ + \angle C = 180^\circ$

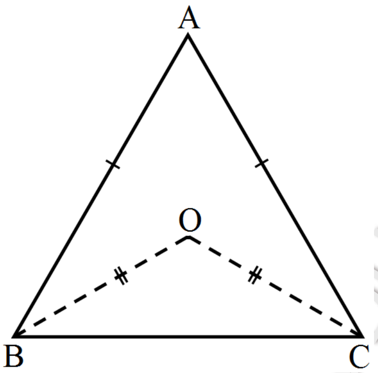
$\Rightarrow \angle C = 80^\circ$

We know that, the greater angle has the longest side opposite to it.

Since $\angle A < \angle B < \angle C$, $BC < AC < AB$.

So, the longest side is AB.

72. In the given figure, AB = AC and OB = OC. Then, $\angle ABO : \angle ACO =?$



a. 1 : 1
b. 2 : 1
c. 1 : 2
d. None of these

Ans. :

a. 1 : 1

Solution:

In $\triangle ABC$,

$AB = AC \Rightarrow \angle ABC = \angle ACB \ldots (i)$

In $\triangle \mathrm{OBC}$,

$\mathrm{OB} = \mathrm{OC} \Rightarrow \angle \mathrm{OBC} = \angle \mathrm{OCB} \ldots \text{(ii)}$

Subtraction (ii) from (i), we get

$\Rightarrow \angle \mathrm{ABO} = \angle \mathrm{ACO}$

So, $\angle \mathrm{ABO} : \angle \mathrm{ACO} = 1 : 1$

73. In $\triangle \mathrm{ABC}$, $\mathrm{BC} = \mathrm{AB}$ and $\angle \mathrm{B} = 80^\circ$. Then, $\angle \mathrm{A} = ?$

a. 50°
b. 40°
c. 100°
d. 80°

Ans. :

a. 50°

Solution:

In $\triangle \mathrm{ABC}$,

$\mathrm{BC} = \mathrm{AB}$

$\Rightarrow \angle \mathrm{A} = \angle \mathrm{C}$ (angles opposite to equal sides are equal)

Now, $\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ$

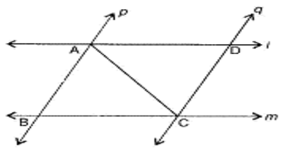
$\Rightarrow \angle \mathrm{A} + 80^\circ + \angle \mathrm{A} = 180^\circ$

$\Rightarrow 2\angle \mathrm{A} + 100^\circ$

$\Rightarrow \angle \mathrm{A} = 50^\circ$

*** Answer the following questions in one sentence. [1 Marks Each] [5]**

74. l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that ΔABC $\cong$ ΔCDA.



Ans. : Given : l || m and p || q

To prove : DABC $\cong$ DCDA

Proof : l || m and p || q . . . . [Given]

In DABC and DCDA

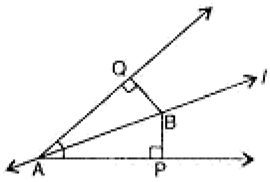
$\angle$BAC = $\angle$DCA . . . . . [Alternate interior angles as AB || DC]

Similarly, $\angle$ACB = $\angle$CAD . . . [Alternate interior angles as BC || DA]

AC = DA . . . [Common]

DABC $\cong$ DCDA [By ASA congruency]

75. Line l is the bisector of an angle $\angle$A and B is any point on l. BP and BQ are perpendicular from B to the arms of $\angle$A.



Show that:

i. $\triangle APB \cong \triangle AQB$

ii. BP = BQ or B is equidistant from the arms of $\angle A$.

Ans. :

i. In $\triangle APB$ and $\triangle AQB$

$\angle BAP = \angle BAQ$ ...[As l bisects $\angle A$]

$\angle BPA = \angle BQA$ ...[Each 90°]

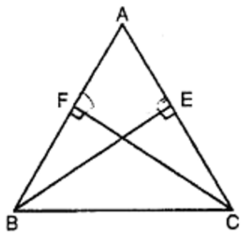
AB = AB ...[Common]

$\therefore \triangle APB \cong \triangle AQB$ proved ...[AAS property] ...(1)

ii. $\triangle APB \cong \triangle AQB$ ...[From (1)]

$\therefore$ BP = BQ ...[c.p.c.t.]

76. ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal.



Ans. : In $\triangle ABE$ and $\triangle ACF$,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC$ = [90°]

AB = AC [Given]

$\therefore \triangle ABE \cong \triangle ACF$ [By ASA congruency]

$\therefore$ BE = CF [By C.P.C.T.]

So Altitudes are equal.

77. Is it possible to construct a triangle with lengths of its sides as given below? Give reason for your answer.

2.5cm, 5cm, 7cm

Ans. : Yes, because the sum of two sides of a triangle is greater than the third side.

$2.5 + 5 > 7$; $5 + 7 > 2.5$; $2.5 + 7 > 5$

78. Is it possible to construct a triangle with lengths of its sides as given below? Give reason for your answer.

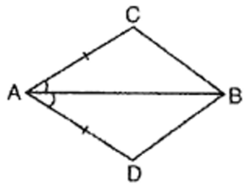
3cm, 4cm, 8cm

Ans. : No, because the sum of two sides of a triangle is not greater than the third side.

$3 + 4 < 8$

* Answer the following short questions. [2 Marks Each] [28]

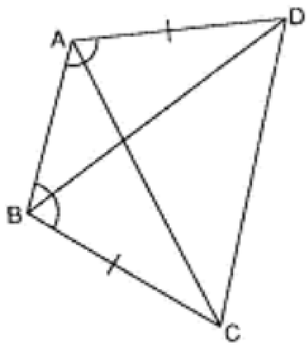
79. In quadrilateral ABCD (See figure). AC = AD and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?



Ans. : Given: In quadrilateral ABCD, AC = AD and AB bisects $\angle A$.
To prove: $\angle ABC \cong \triangle ABD$
Proof: In $\triangle ABC$ and $\triangle ABD$,
AC = AD [Given]
$\angle BAC = \angle BAD$ [$\because$ AB bisects $\angle A$]
AB = AB [Common]
$\therefore \triangle ABC \cong \triangle ABD$ [By SAS congruency]
Thus BC = BD [By C.P.C.T.]

80. ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$: Prove that:

i. $\triangle ABD \cong \triangle BAC$
ii. BD = AC
iii. $\angle ABD = \angle BAC$



Ans. : In quadrilateral ACBD, we have AD = BC and $\angle DAB = \angle CBA$

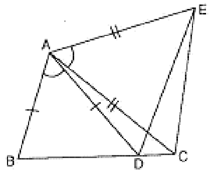
i. In ΔABC and ΔBAC,
AD = BC (Given)
$\angle DAB = \angle CBA$ (Given)
AB = AB (Common)
$\triangle ABD \cong \triangle BAC$ ...[By SAS Congruence]

ii. Since $\Delta ABD \cong \Delta BAC$
$\Rightarrow$ BD = AC [By C.P.C.T.]

iii. Since $\Delta ABD \cong \Delta BAC$
$\Rightarrow \angle ABD = \angle BAC$ [By C.P.C.T.]

81.

In figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



Ans. : Given : AC = AE, AB = AD and ∠BAD = ∠EAC.

To prove ; BC = DE

Proof : In DABC and DADE

AC = AE, AB = AD and ∠BAD = ∠EAC ...[Given]

∴ ∠BAD + ∠DAC = ∠DAC + ∠EAC ...[Adding ∠DAC to both sides]

∴ ∠BAC = ∠DAE ...(1)

AC = AE ...[Given]

∠BAC = ∠DAE ...[From (1)]

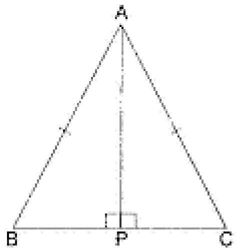
AB = AD ...[Given]

∴ DABC ≅ DADE ...[By SAS property]

∴ BC = DE ...[c.p.c.t.]

82. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

Ans. :



Given: ABC is an isosceles triangle with AB = AC.

To Prove : ∠B = ∠C

Construction: Draw AP ⊥ BC

Proof: In right triangle APB and right triangle APC.

AB = AC . . . . [given]

AP = AP . . . .[Common]

∴ △APB ≅ △APC . . . [RHS rule]

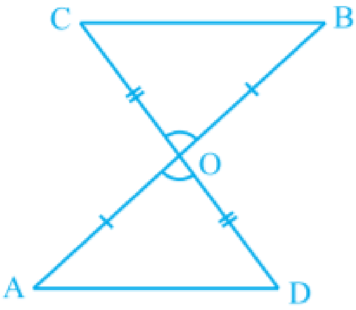
∴ ∠ABP = ∠ACP . . .[c.p.c.t.]

∴ ∠B = ∠C

83. In Fig., OA = OB and OD = OC. Show that

i. △ AOD ≅ △ BOC

ii. AD || BC

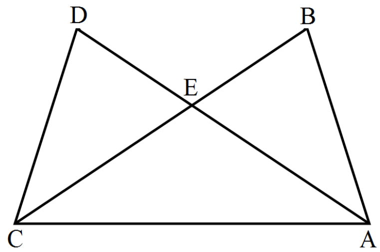


Ans. :

i. You may observe that in △ AOD and △ BOC,
OA = OB (Given)
OD = OC
Also, since ∠ AOD and ∠ BOC form a pair of vertically opposite angles,
we have ∠ AOD = ∠ BOC
So, △ AOD ≅ △ BOC (by the SAS congruence rule)

ii. In congruent triangles AOD and BOC, the other corresponding parts are also equal.
So, ∠ OAD = ∠ OBC and these form a pair of alternate angles for line segments AD and BC.
Therefore, AD || BC

84. In Fig. it is given that $\mathrm{AB} = \mathrm{CD}$ and $\mathrm{AD} = \mathrm{BC}$. Prove that $\triangle\mathrm{ADC} \cong \triangle\mathrm{CBA}$.



Ans. : In $\triangle\mathrm{ADC}$ and $\triangle\mathrm{CBA}$

$\mathrm{AB} = \mathrm{CD}$ (given)

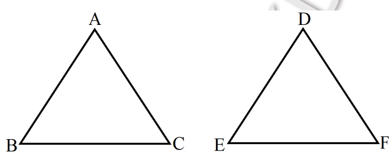
$\mathrm{AC} = \mathrm{AC}$ (common)

$\mathrm{AD} = \mathrm{BC}$ (given)

By SSS congurence criterion $\triangle\mathrm{ADC} \cong \triangle\mathrm{CBA}$

85. In two triangles ABC and DEF, it is given that $\angle\mathrm{A} = \angle\mathrm{D}, \angle\mathrm{B} = \angle\mathrm{E}$ and $\angle\mathrm{C} = \angle\mathrm{F}$, Are the two triangles necessarily congruent?

Ans. : It is given that $\angle\mathrm{A} = \angle\mathrm{D}, \angle\mathrm{B} = \angle\mathrm{E}, \angle\mathrm{C} = \angle\mathrm{F}$



For necessarily trialnge to be congruent, sides also be equal.

86. In $\triangle\mathrm{ABC}$, $\angle\mathrm{A} = 100^{\circ}$ and $\angle\mathrm{C} = 50^{\circ}$. Which is its shortest side?

Ans. : Given: $\angle\mathrm{A} = 100^{\circ}$ and $\angle\mathrm{C} = 50^{\circ}$

Using angle sum property of triangle,

$\angle B = 30^\circ$

Since, $\angle A$ is the greatest angle.

So, the shortest side is BC.

87. In $\triangle ABC$, $\angle A = \angle B = 45^\circ$. Which is its longest side?

Ans. : Given: $\angle A = \angle B = 45^\circ$

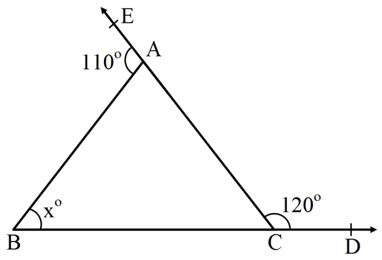
Using angle sum property of triangle,

$\angle C = 90^\circ$

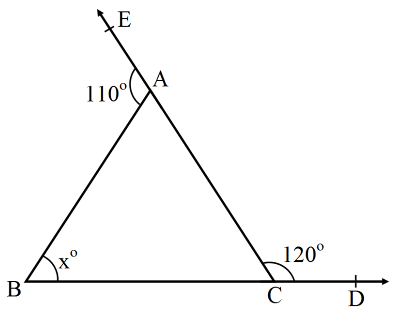
Since, $\angle C$ is the greatest angle.

So, the longest side is AB.

88. Calculate the value of x in the following figures.



Ans. : $\angle EAB + \angle BAC = 180^\circ$ [Linear pair angles]



$110^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 110^\circ = 70^\circ$

Again, $\angle BCA + \angle ACD = 180^\circ$ [Linear pair angles]

$\Rightarrow \angle BCA + 120^\circ = 180^\circ$

$\Rightarrow \angle BCA = 180^\circ - 120^\circ = 60^\circ$

Now, in $\triangle ABC$,

$\angle ABC + \angle BAC + \angle ACB = 180^\circ$

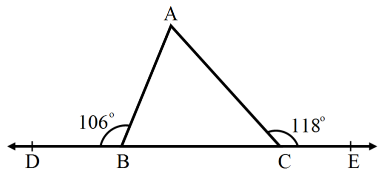
$x + 70^\circ + 60^\circ = 180^\circ$

$\Rightarrow x + 130^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 130^\circ = 50^\circ$

$\therefore x = 50$

89. In the given figure, the side BC of $\triangle ABC$ has been produced on both sides-on the left to D and on the right to E. If $\angle ABD = 106^\circ$ and $\angle ACE = 118^\circ$, find the measure of each angle of the triangle.



Ans. : As $\angle DBA$ and $\angle ABC$ form a linear pair.

So, $\angle DBA + \angle ABC = 180^\circ$

$\Rightarrow 106^\circ + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 106^\circ = 74^\circ$

Also, $\angle ACB$ and $\angle ACE$ form a linear pair.

So, $\angle ACB + \angle ACE = 180^\circ$

$\Rightarrow \angle ACB + 118^\circ = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 118^\circ = 62^\circ$

In $\triangle ABC$, we have,

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$

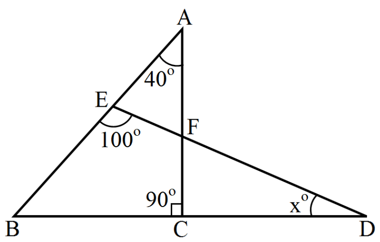
$74^\circ + 62^\circ + \angle BAC = 180^\circ$

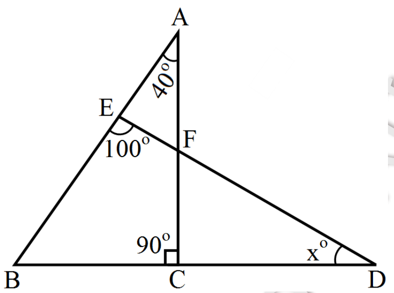
$\Rightarrow 136^\circ + \angle BAC = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 136^\circ = 44^\circ$

$\therefore$ In triangle ABC, $\angle A = 44^\circ, \angle B = 74^\circ$ and $\angle C = 62^\circ$

90. Calculate the value of x in the following figures.





Ans. :

In $\triangle AEF$,

Exterior $\angle BED = \angle EAF = \angle EFA$

$\Rightarrow 100^\circ = 40^\circ + \angle EFA$

$\Rightarrow \angle EFA = 100^\circ - 40^\circ = 60^\circ$

Also, $\angle CFD = \angle EFA$ [Vertically Opposite angles]

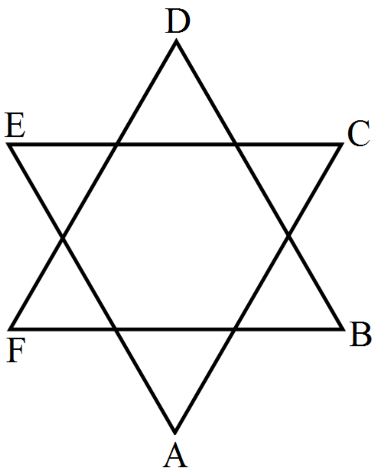
$\Rightarrow 90^\circ = 60^\circ + x^\circ$

$\Rightarrow x^\circ = 90^\circ - 60^\circ = 30^\circ$

$\therefore x = 30$

91. In the adjoining figure, show that

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^\circ$.



Ans. : In $\triangle \mathrm{ACE}$, we have,

$\angle \mathrm{A} + \angle \mathrm{C} + \angle \mathrm{E} = 180^\circ \ \ldots \text{(i)}$

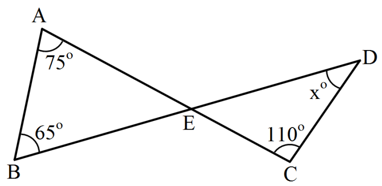
In $\triangle \mathrm{BDF}$, we have,

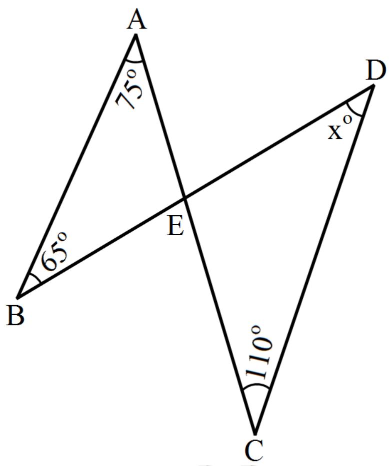
$\angle \mathrm{B} + \angle \mathrm{D} + \angle \mathrm{F} = 180^\circ \ \ldots \text{(ii)}$

Adding both sides of (i) and (ii), we get,

$\angle \mathrm{A} + \angle \mathrm{C} + \angle \mathrm{E} + \angle \mathrm{B}$
$+ \angle \mathrm{D} + \angle \mathrm{F} = 180^\circ + 180^\circ$
$\Rightarrow \angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} + \angle \mathrm{D} + \angle \mathrm{E} + \angle \mathrm{F} = 360^\circ.$

92. Calculate the value of x in the following figures.





Ans. :

In $\triangle \mathrm{ABE}$, we have,

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{E} = 180^\circ$

$\Rightarrow 75^\circ + 65^\circ + \angle \mathrm{E} = 180^\circ$

$\Rightarrow 140^\circ + \angle \mathrm{E} = 180^\circ$

$\Rightarrow \angle \mathrm{E} = 180^\circ - 140^\circ = 40^\circ$

Now, $\angle \mathrm{CED} = \angle \mathrm{AEB}$ [Vertically opposite angles]

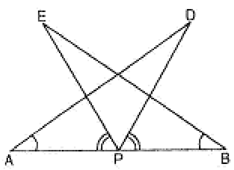
$\Rightarrow \angle \mathrm{CED} = 40^\circ$

Now, in $\triangle \mathrm{CED}$, we have,

$\angle C + \angle E + \angle D = 180^\circ$
$\Rightarrow 110^\circ + 40^\circ + x^\circ = 180^\circ$
$\Rightarrow 150^\circ + x^\circ = 180^\circ$
$\Rightarrow x^\circ = 180^\circ - 150^\circ = 30^\circ$
$\therefore x = 30$

*** Answer the following questions. [3 Marks Each] [42]**

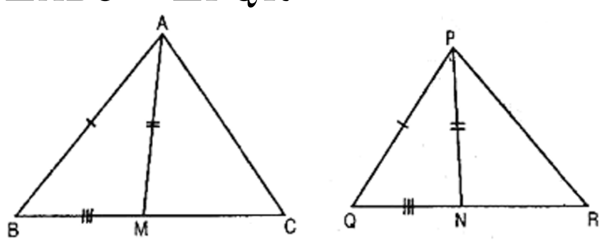
93. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB.
Show that:
   i. △DAP ≅ △EBP
   ii. AD = BE



Ans. : Given: AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB.
To prove:
   i. DDAP ≅ DEBP
   ii. AD = BE

Proof :(ii)
∠EPA = ∠DPB ...[Given]
∠EPA + ∠EPD = ∠EPD + ∠DPB ...[Adding ∠EPD to both sides]
∠APD = ∠BPE ...(1)
In DDAP and DEBP
∠DAP = ∠EBP ...[Given]
AP = BP ...[As P is the mid-point of the line AB]
∠APD = ∠BPE ...[From (1)]
∴ DDAP ≅ DEBP proved ...[ASA property] ...(2)
(i) As DDAP ≅ DEBP ...[From (2)]
∴ AD = BE ...[c.p.c.t.]

94. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of PQR (See figure). Show that:
   i. $\triangle ABM \cong \triangle PQN$
   ii. $\triangle ABC \cong \triangle PQR$



Ans. : AM is the median of $\triangle$ABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC ...(i)

PN is the median of $\triangle$PQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR ...(ii)

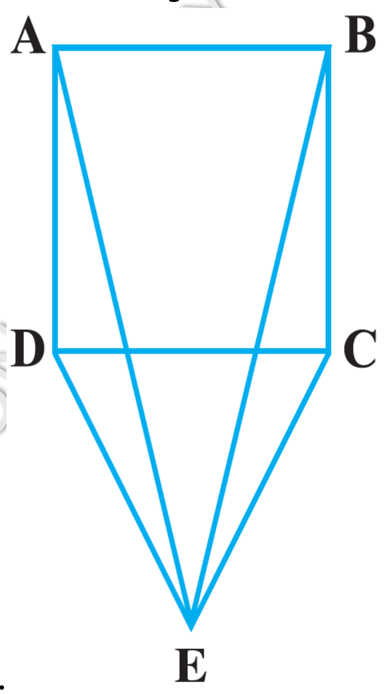
Now BC = QR [Given] $\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$

$\therefore$ BM = QN ...(iii)

i. Now in $\triangle$ABM and $\triangle$PQN,
AB = PQ[Given]
AM = PN[Given]
BM = QN[From eq.(iii)]
$\therefore$ $\triangle$ABM $\cong$ $\triangle$PQN [By SSS congruency]
$\Rightarrow$ $\angle$B = $\angle$Q [By C.P.C.T.] ...(iv)

ii. In $\triangle$ABC and $\triangle$PQR,
AB = PQ [Given]
$\angle$B = $\angle$Q [Prove above]
$\therefore$ PR = QR [Given]
ABC $\cong$ PQR [By SAS congruency]

95. In the given figure $\triangle \mathrm{CDE}$ is an equilateral triangle formed on a side CD of a square



ABCD. Show that $\triangle \mathrm{ADE} \cong \triangle \mathrm{BCE}$.

Ans. : Given, $\triangle \mathrm{CDE}$ is an equilateral triangle formes on a side CD of a ABCD.

$\triangle \mathrm{ADE} \cong \triangle \mathrm{BCE}$

In $\triangle \mathrm{ADE}$ and $\triangle \mathrm{BCE}$,

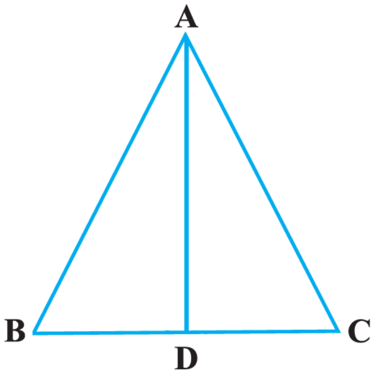
$\mathrm{DE} = \mathrm{CE}$

$\angle \mathrm{ADE} = \angle \mathrm{BCE}$

$\mathrm{AD} = \mathrm{BC}$

$\triangle \mathrm{ADE} \cong \triangle \mathrm{BCE}$

96. ABC is an isosceles triangle with AB = AC and D is a point on BC such that $\mathrm{AD} \perp \mathrm{BC}$ (see figure). To prove that $\angle \mathrm{BAD} = \angle \mathrm{CAD}$, a student proceeded as



follows: In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$

$\angle B = \angle C$

$\angle ADM = \angle ADC$

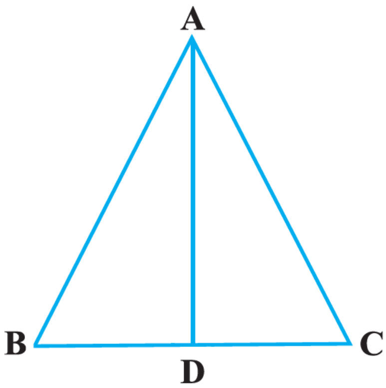
$\therefore \triangle ABD \cong \triangle ADC$

$\angle BAD = \angle CAD$

What is the defect in the above arguments?

Ans. : In $\triangle ABC$,

$AB = AC$



$\Rightarrow \angle ACB = \angle ABC$ In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$

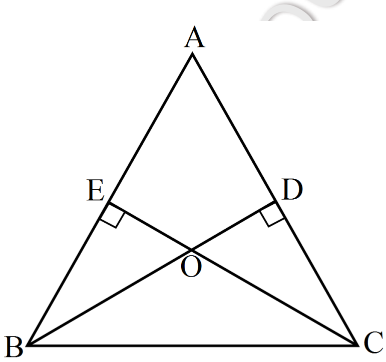
$\angle ADB = \angle ACD$

$\triangle ABD \cong \triangle ACB$

So, the defect in the given argument is that firstiy prove $\triangle ABD \cong \triangle ACB$.

97. In a $\triangle$ ABC, $BD \perp AC$ and $CE \perp AB$. If BD and CE intersect O, prove that $\angle BOC = 180^\circ - A$.

Ans. :



In quadrilateral AEOD

$\angle A + \angle AEO + \angle EOD + \angle ADO = 360^\circ$

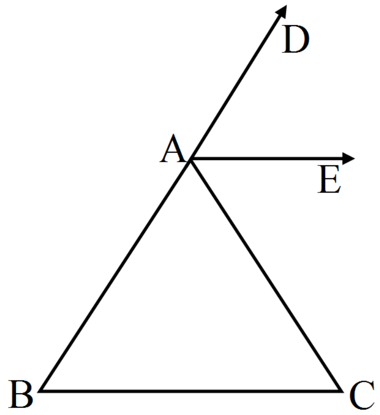
$\Rightarrow \angle A + 90^\circ + 90^\circ + \angle EOD = 360^\circ$

$\Rightarrow \angle A + \angle BOC = 180^\circ$ [$\therefore \angle EOD = \angle BOC$ vertically opposite angles]
$\Rightarrow \angle BOC = 180^\circ - \angle A$

98. The angles of a triangle are (x - 40)º, (x - 20)º and $\left(\frac{1}{2}x - 10\right)^\circ$. Find the value of x.

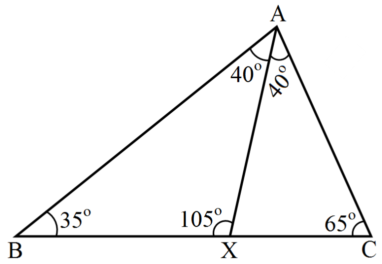
Ans. : Given that,
The angles of a triangle are
$(x - 40^\circ), (x - 20^\circ)$ and $\left(\frac{1}{2}x - 10^\circ\right)$
We know that,
Sum of all angles of triangle is 180°
$\therefore (x - 40^\circ) + (x - 20^\circ) + \left(\frac{1}{2}x - 10^\circ\right) = 180^\circ$
$2x + \frac{1}{2}x - 70^\circ = 180^\circ$
$\frac{5}{2}x = 180^\circ + 70^\circ$
$5x = 2(250)^\circ$
$x = \frac{500^\circ}{5}$
$\therefore x = 100^\circ$

99. In figure AE bisects $\angle CAD$ and $\angle B = \angle C$. Prove that AE || BC.

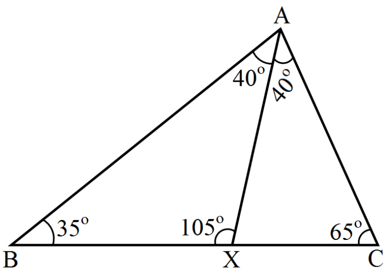


Ans. : Let $\angle B = \angle C = x$
Then,
$\angle CAD = \angle B + \angle C = 2x$ (exterior angle)
$\Rightarrow \frac{1}{2}\angle CAD = x$
$\Rightarrow \angle EAC = x$
$\Rightarrow \angle EAC = \angle C$
These are alternate interior angles for the lines AE and BC
$\therefore AE \,||\, BC$

100. In $\triangle ABC$, $\angle B = 35^\circ, \angle C = 65^\circ$ and the bisector of $\angle BAC$ meets BC in X. Arrange AX, BX and CX in descending order.



Ans. :



Given: in $\triangle ABC, \angle B = 35^\circ, \angle C = 65^\circ$ and the bisector of $\angle BAC$ meets BC in x

In $\triangle ABX,$

$\because \angle BAX > \angle ABX$

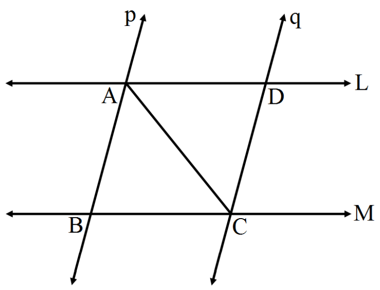
$\therefore BX > AX \ldots (i)$

$\therefore AX > CX \ldots (ii)$

From (i) and (ii), we get

$BX > AX > CX$

101. In the given figure, two parallel line l and m are intersected by two parallel lines p and q.
Show that $\triangle ABC \cong \triangle CDA$.



Ans. : In $\triangle ABC$ and $\triangle CDA$

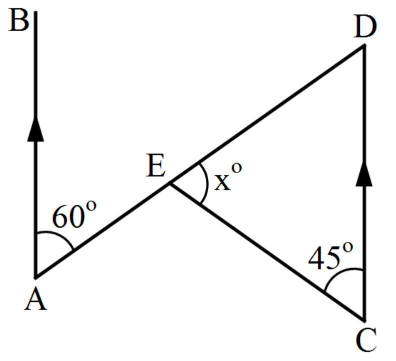
$\angle BAC = \angle DCA$ (alternate interior angles for p || q)

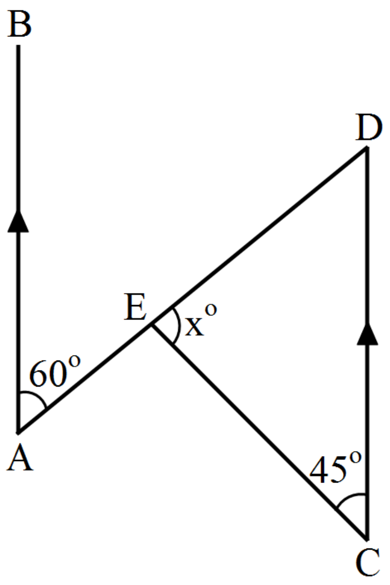
$AC = CA$ (common)

$\angle BCA = \angle DAC$ (Alternate interior angles for l || m)

$\therefore \triangle ABC \cong \triangle CDA$ (by ASA congruence rule)

102. Calculate the value of x in the following figures.





Ans. :

Since AB || CD and AD is a transversal.

So, $\angle BAD = \angle ADC$

$\Rightarrow \angle ADC = 60^\circ$

In $\triangle ECD$, we have,

$\angle E + \angle C + \angle D = 180^\circ$

$\Rightarrow x^\circ + 45^\circ + 60^\circ = 180^\circ$

$\Rightarrow x^\circ + 105^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ - 105^\circ = 75^\circ$

$\therefore x = 75$

103. In $\triangle ABC$, if $\angle A + \angle B = 125^\circ$ and $\angle A + \angle C = 113^\circ$, find $\angle A, \angle B$ and $\angle C$.

Ans. : Since. $\angle A, \angle B$ and $\angle C$ are the angles of a triangle .

So, $\angle A + \angle B + \angle C = 180^\circ$

Now, $\angle A + \angle B = 125^\circ$ [Given]

$\therefore 125^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ = 125^\circ = 55^\circ$

Also, $\angle A + \angle C = 113^\circ$ [Given]

$\Rightarrow \angle A + 55^\circ = 113^\circ$

$\Rightarrow \angle A = 113^\circ - 55^\circ = 58^\circ$

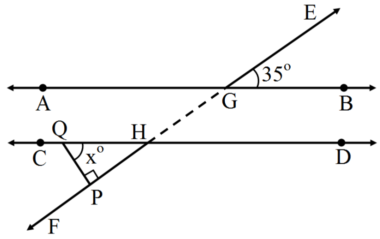
Now as $\angle A + \angle B = 125^\circ$

$\Rightarrow 58^\circ + \angle B = 125^\circ$

$\Rightarrow \angle B = 125^\circ - 58^\circ = 67^\circ$

$\therefore \angle A = 58^\circ, \angle B = 67^\circ$ and $\angle C = 55^\circ$.

104. In the given figure, AB || CD and EF is a transversal, cutting them at G and H respectively. If $\angle EGB = 35^\circ$ and $QP \perp EF$, find the measure of $\angle PQH$.



Ans. : AB || CD and EF is the transversal.

$\Rightarrow \angle EGB = \angle GHD$ (corresponding angles)

$\angle GHD = 35^\circ$

Now, $\angle GHD = \angle QHP$ (vertically opposite angles)

$\Rightarrow \angle QHP = 35^\circ$

In DQHP, by angle sum property,

$\angle PQH + \angle QHP + \angle QPH = 180^\circ$

$\Rightarrow \angle PQH + 35^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle PQH = 55^\circ$

105. In $\triangle ABC$, if $\angle A + \angle B = 108^\circ$ and $\angle B + \angle C = 130^\circ$, find $\angle A, \angle B$ and $\angle C$.

Ans. : $\angle A + \angle B = 108^\circ$ [Given]

But as $\angle A, \angle B$ and $\angle C$ are the angles of a triangle,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 108^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 108^\circ = 72^\circ$

Also, $\angle B + \angle C = 130^\circ$ [Given]

$\Rightarrow \angle B + 72^\circ = 130^\circ$

$\Rightarrow \angle B + 72^\circ = 130^\circ$

$\Rightarrow \angle B = 130^\circ - 72^\circ = 58^\circ$

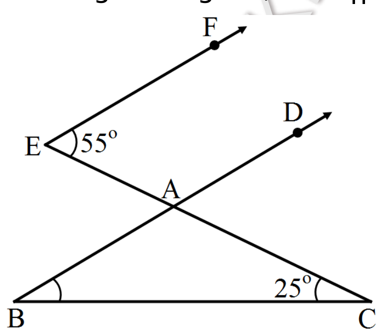
Now as, $\angle A + 58^\circ = 108^\circ$

$\Rightarrow \angle A + 58^\circ = 108^\circ$

$\Rightarrow \angle A = 108^\circ - 58^\circ = 50^\circ$

$\therefore \angle A = 50^\circ, \angle B = 58^\circ$ and $\angle C = 72^\circ$

106. In the given figure, BAD || EF, $\angle AEF = 55^\circ$ and $\angle ACB = 25^\circ$, find $\angle ABC$.



Ans. : BAD || EF and EC is the transversal.

$\Rightarrow \angle\mathrm{AEF} = \angle\mathrm{CAD}$ (corresponding angles)

$\Rightarrow \angle\mathrm{CAD} = 55^\circ$

Now, $\angle\mathrm{CAD} + \angle\mathrm{CAB} = 180^\circ$ (linear pair)

$\Rightarrow 55^\circ + \angle\mathrm{CAB} = 180^\circ$

$\Rightarrow \angle\mathrm{CAB} = 125^\circ$

In $\triangle\mathrm{ABC}$, by angle sum property,

$\angle\mathrm{ABC} + \angle\mathrm{CAB} + \angle\mathrm{ACB} = 180^\circ$

$\Rightarrow \angle\mathrm{ABC} + 125^\circ + 25^\circ = 180^\circ$

$\Rightarrow \angle\mathrm{ABC} = 30^\circ$

* Questions with calculation. [4 Marks Each] [80]

107. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

Ans. : We have to prove that $\angle\mathrm{ABC} = 90^\circ$

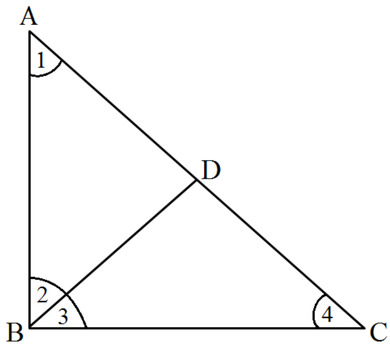
As is the mid-point of AC,

so, AD = DC

Also, $\mathrm{BD} = \frac{1}{2}\mathrm{AC} = \mathrm{AD}$

$\left[\therefore\right.$ D is the mid-point of AC$\left.\right]$

$\therefore$ BD = AD = DC



In $\triangle\mathrm{ABC}$, We have

BD = AD

$\therefore \angle 1 = \angle 2 \ldots (1)$

$\left[\because\right.$ Angles opposite to equal sides are equal$\left.\right]$

In $\triangle\mathrm{BCD}$,We have

BD = DC,

$\therefore \angle 3 = \angle 4 \ldots (2)$

In $\triangle\mathrm{ABC}$, We have

$\angle 1 + \angle\mathrm{ABC} + \angle 4 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 \angle + 4 = 180^\circ$

$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ)$

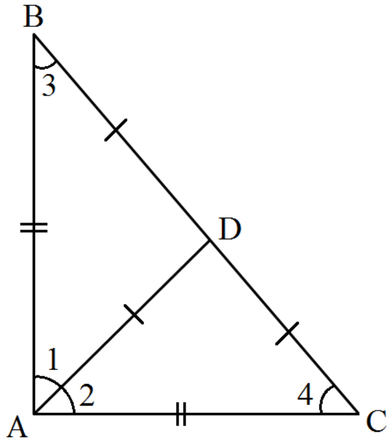
$\Rightarrow \angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle = 90^\circ$

Hence proved.

108. ABC is a right triangle with AB = AC. If bisector of meets BC at D,then prove that BC = 2AD.

Ans. : In $\triangle \mathrm{ABC}$ is a right angle triangle with AB = AC, is the Bisector.



$\mathrm{BC}=2\mathrm{AD}$

$\mathrm{AB}=\mathrm{AC}$

$\angle \mathrm{C}=\angle \mathrm{B}$

Now, in right angle $\triangle \mathrm{ABC}$,

$\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}$

$\Rightarrow 90^{\circ}+\angle \mathrm{B}+\angle \mathrm{B}=180^{\circ}$

$\Rightarrow 2\angle \mathrm{B}=90^{\circ}$

$\Rightarrow \angle \mathrm{B}=45^{\circ}$

$\Rightarrow \angle \mathrm{B}=\angle \mathrm{C}=45^{\circ}$

$\Rightarrow \angle 3=\angle 4=45^{\circ}$

$\Rightarrow \angle 1=\angle 2=45^{\circ}$

$\mathrm{BD}=\mathrm{AD}, \mathrm{DC}=\mathrm{AD}$

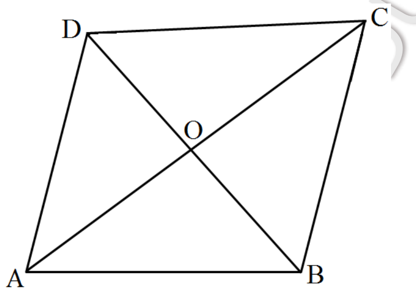
Hence, $\mathrm{BC}=\mathrm{BD}+\mathrm{CD}$

$=\mathrm{AD}+\mathrm{AD}$

$=2\mathrm{AD}$

109. Show that in a quadrilateral ABCD, AB + BC + CD + DA < 2 (BD + AC)

Ans. : Given, A quadrilateral ABCD.



To prove: AB + BC + CD + DA < 2(BD + AC)

Proof: In $\triangle \mathrm{AOD}$ We have

$\therefore \mathrm{OA}+\mathrm{OB}>\mathrm{AB}\ldots(\mathrm{i})$

$\left[\therefore\right.$ Sum of the lengths of any sides of a triangle must be greater than rhird side$\left.\right]$

In $\triangle \mathrm{BOC}$ We have

OB + OC + > BC ...(2) [Same reason]

In $\triangle \mathrm{COD}$, We have

OC + OD + > CD ...(3) [Same reason]

In $\triangle \mathrm{DOA}$, We have

OD + OA > DA ...(4) [Same reason]

Adding (1), (2), (3) and (4), We get

OA + B + OB + OC + OC + OD + OA + > AB + BC + CD +DA

⇒ 2(OA + OB + OC + OD) > AB + BC + CD + DA

⇒ 2{(OA + OC) + (OB + OD)} > AB + BC + CD + DA

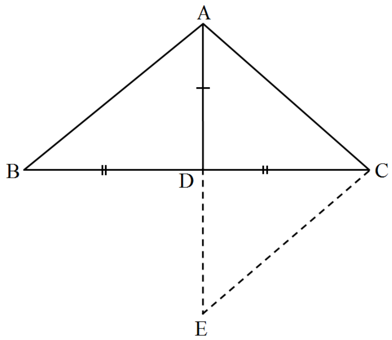
⇒ 2(AC + BD) > AB + BC + CD + DA

⇒ AB + BC + CD + DA < 2(BD + AC)

Hence, proved.

110. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Ans. : Givan in $\triangle \mathrm{ABC}$, AD is a median.



Construction produce AD to a point E such that AD = DE and join CE.

To prove AC + AB > 2AD

Proof in $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ECD}$,

$\mathrm{AD} = \mathrm{DE}$ [by construction]

$\mathrm{BD} = \mathrm{CD}$ [given AD is the media]

and $\angle \mathrm{ADB} = \angle \mathrm{CDE}$ [vertically opposite angle]

$\therefore\ \triangle \mathrm{ABD} \cong \mathrm{ECD}$ [by SAS congruence rule]

$\Rightarrow\ \mathrm{AB} = \mathrm{CE}$ [by CPCT]...(i)

Now, in $\triangle \mathrm{AEC}$,

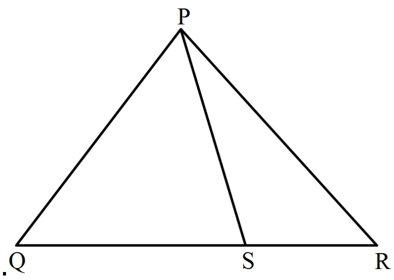
$\mathrm{AC} + \mathrm{EC} > \mathrm{AE}$

$\therefore\ \mathrm{AC} + \mathrm{AB} > 2\mathrm{AD}$ [Sum of two sides of a triangle is greater than the third side]

[from Eq. (i) and also taken that AD = DE]

Hence proved.

111. S is any point on side QR of a $\triangle \mathrm{PQR}$. show that PQ + QR + RP > 2PS.



Ans. : Given A point S on QR. In $\triangle PQS$, we have

$\mathrm{PQ}+\mathrm{QS}>\mathrm{PS}\ldots(\mathrm{i})$

Now, in $\triangle PSR$, we have

$\mathrm{RS}+\mathrm{RP}>\mathrm{PS}\ldots(\mathrm{ii})$

From eq.(i) and (ii),

$\mathrm{PQ}+\mathrm{QS}+\mathrm{RS}+\mathrm{RP}>2\mathrm{PS}$

$\Rightarrow \mathrm{PQ}+\mathrm{QR}+\mathrm{RP}>2\mathrm{PS}$

112. In a triangle ABC, D is the mid-point of side AC such that $\mathrm{BD}=\frac{1}{2}\mathrm{AC}$. Show that $\angle\mathrm{ABC}$ is a right angle.

Ans. : We have to prove that $\angle\mathrm{ABC}=90^{\circ}$

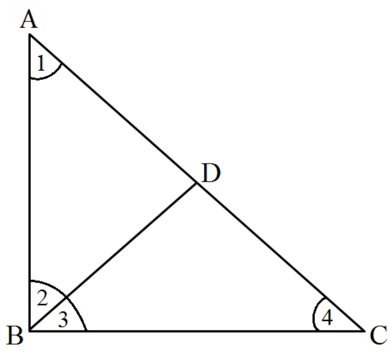
As is the mid-point of AC,

so, AD = DC

Also, $\mathrm{BD}=\frac{1}{2}\mathrm{AC}=\mathrm{AD}$

$\left[\therefore\right.$ D is the mid-point of AC$\left.\right]$

$\therefore$ BD = AD = DC



In $\triangle ABC$, We have

BD = AD

$\therefore \angle 1=\angle 2\ldots(1)$

$\left[\because\right.$ Angles opposite to equal sides are equal$\left.\right]$

In $\triangle BCD$, We have

BD = DC,

$\therefore \angle 3=\angle 4\ldots(2)$

In $\triangle ABC$, We have

$\angle 1+\angle\mathrm{ABC}+\angle 4=180^{\circ}$

$\Rightarrow \angle 1+\angle 2+\angle 3\angle+4=180^{\circ}$

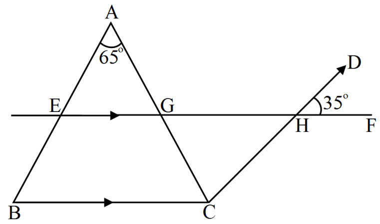
$\Rightarrow 2(\angle 2+\angle 3)=180^{\circ})$

$\Rightarrow \angle 2+\angle 3=90^{\circ}$

$\Rightarrow \angle = 90^\circ$

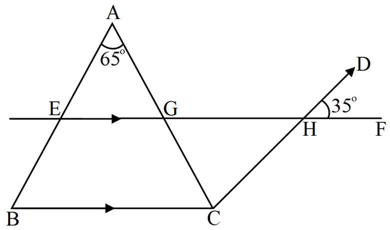
Hence proved.

113. In the given figure, if $\mathrm{AB} \,||\, \mathrm{CD}, \mathrm{EF} \,||\, \mathrm{BC}, \angle\mathrm{BAC} = 65^\circ$ and $\angle\mathrm{DHF} = 35^\circ$, find $\angle\mathrm{AGH}$.



Ans. : In the given figure, if $\mathrm{AB} \,||\, \mathrm{CD}, \mathrm{EF} \,||\, \mathrm{BC}, \angle\mathrm{BAC} = 65^\circ$ and $\angle\mathrm{DHF} = 35^\circ$

We need to find $\angle\mathrm{AGH}$.



Here, GF and CD are straight lines intersecting at point H, so using the property, "vertically opposite angles are equal", we get,

$\angle\mathrm{DHF} = \angle\mathrm{GHC}$

$\angle\mathrm{GHC} = 35^\circ$

Further, as AB || CD and AC is the transversal

Using the property, "alternate interior angles are equal"

$\angle\mathrm{BAC} = \angle\mathrm{ACD}$

$\angle\mathrm{BAC} = 65^\circ$

Further applying angle sum property of the triangle

In $\triangle\mathrm{GHC}$

$\angle\mathrm{DHF} + \angle\mathrm{HCG} + \angle\mathrm{CGH} = 180^\circ$

$\angle\mathrm{CGH} + 35^\circ + 65^\circ = 180^\circ$

$100^\circ + \angle\mathrm{CGH} = 180^\circ$

$\angle\mathrm{CGH} = 180^\circ - 100^\circ$

$\angle\mathrm{CGH} = 80^\circ$

Hence, applying the property, "angles forming a linear pair are supplementary"

As AGC is a straight line

$\angle\mathrm{CGH} + \angle\mathrm{AGH} = 180^\circ$

$\angle\mathrm{AGH} + 80^\circ = 180^\circ$

$\angle\mathrm{AGH} = 180^\circ - 80^\circ$

$\angle\mathrm{AGH} = 100^\circ$

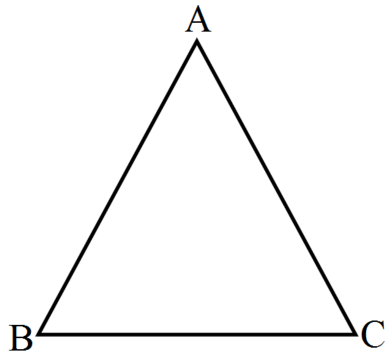
Therefore,$\angle\mathrm{AGH} = 100^\circ$

114. If the angles A, B and C of $\triangle\mathrm{ABC}$ satisfy the relation B − A = C − B, then find the measure of $\angle\mathrm{B}$.

Ans. : In the given $\triangle\mathrm{ABC}$,

$\angle A, \angle B$ and $\angle C$ satisfy the relation $B - A = C - B$

We need to fine the measure of $\angle B$.



As,

$B - A = C - B$

$B + B = C + A$

$2B = C + A$

$2B - A = C \ldots (1)$

Now, using the angle sum property of the triangle, we get,

$A + B + C = 180^\circ$

$2B - A + A + \angle B = 180^\circ (\text{Using } 1)$

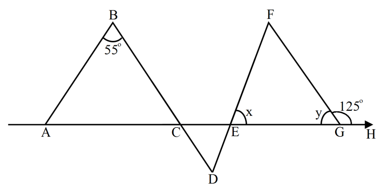
$3B = 180^\circ$

$B = \frac{180^\circ}{3}$

$= 60^\circ$

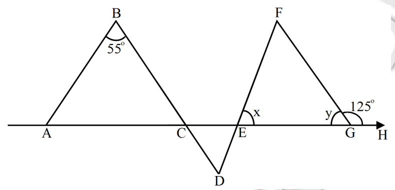
Therefore, $\angle B = 60^\circ$

115. In the given figure, if $AB \parallel DE$ and $BD \parallel FG$ such that $\angle FGH = 125^\circ$ and $\angle B = 55^\circ$, find x and y.



Ans. : In the given figure, if $AB \parallel DE, BD \parallel FG, \angle FGH = 125^\circ$ and $\angle B = 55^\circ$

We need to find the value of *x* and *y*



Here, as AB || DE and BD is the transversal, so according to the property, "alternate interior angles are equal", we get

$\angle D = \angle B$

$\angle D = 55^\circ \ldots (1)$

Similarly, as BD || FG and DF is the transversal

$\angle D = \angle F$

$\angle F = 55^\circ (\text{Using } 1)$

Further, EGH is a straight line. So, using the property, angles forming a linear pair are supplementary

$\angle FGE = \angle FGH = 180^\circ$

$y + 125^\circ = 180^\circ$

$y = 180^\circ - 125^\circ$

$y = 55^\circ$

Also, using the property, "an exterior angle of a triangle is equal to the sum of the two opposite interior angles", we get,

In $\triangle EFG$ with $\angle FGH$ as its exterior angle

$ext.\angle FGH = \angle F + \angle E$

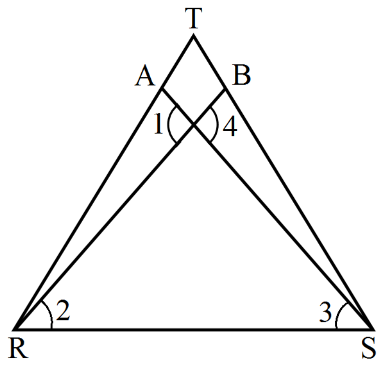
$125^\circ = 55^\circ + x$

$x = 125^\circ - 55^\circ$

$x = 70^\circ$

Thus, $x = 70^\circ$ and $y = 55^\circ$

116. In Fig. it is given that $RT = TS, \angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$. prove that $\triangle RBT \cong \triangle SAT$.



Ans. : Here, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$ $\left[\because \text{Exterior angle = sum of opposite interior angles}\right]$

$\angle 1 = \angle 4$ [vertically opposite angle]

$\therefore 2\angle 2 = 2\angle 3$

$\Rightarrow \angle 2 = \angle 3$

Now $RT = TS$ [given]

$\Rightarrow \angle TRS = \angle TSR$ [Angle opposite to equal sides are equal]

$\therefore \angle TRS - \angle 2 = \angle TSR - \angle 3$

$\Rightarrow \angle TRB = \angle TSA$

Now in $\triangle RBT$ and $\triangle SAT$

$\angle T = \angle T$ [common]

$\angle TRB = \angle TSA$ [proved earlier]

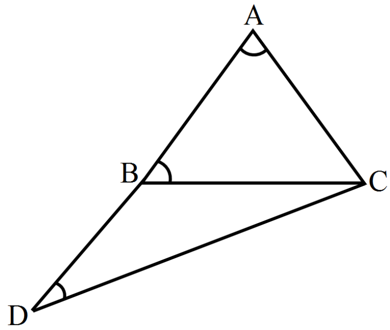
$RT = TS$ [given]

By ASA conguence criterion $\triangle RBT \cong \triangle SAT$

117. In $\triangle ABC$, side AB is produced to D so that $BD = BC$. if $\angle B = 60^\circ$ and $\angle A = 70^\circ$. Prove that: (i) AD > CD (ii) AD > AC

Ans. : Given that, in $\triangle ABC$, side AB is produced to D so that $BD = BC$.

$\angle B=60^{\circ}$, and $\angle A=70^{\circ}$



To prove,

AD > CD AD > AC

First join C and D

We know that,

Sum of angles in a triangle =180°

$\angle A+\angle B+\angle C=180^{\circ}$

$70^{\circ}+60^{\circ}+\angle C=180^{\circ}$

$\angle C=180^{\circ}-(130^{\circ})=50^{\circ}$

$\angle C=50^{\circ}$

$\angle ACB=50^{\circ}\ \ldots(i)$

And also in $\triangle BDC$

$\angle DBC=180^{\circ}-\angle ABC$ [ABD is a straight angle]

$180^{\circ}-60^{\circ}=120^{\circ}$

and also $BD=BC$ [given]

$\angle BCD=\angle BDC$ [Angles opposite to equal sides are equal]

Now,

$\angle DBC+\angle BCD+\angle BDC=180^{\circ}$ [Sum of angles in a triangle =180°]

$\Rightarrow 120^{\circ}+\angle BCD+\angle BCD=180^{\circ}$

$\Rightarrow 120^{\circ}+2\angle BCD=180^{\circ}$

$\Rightarrow 2\angle BCD=180^{\circ}-120^{\circ}=60^{\circ}$

$\Rightarrow \angle BCD=30^{\circ}$

$\Rightarrow \angle BCD=\angle BDC=30^{\circ}\ \ldots.(ii)$

Now, consider $\triangle ADC$.

$\angle BAC\Rightarrow\angle DAC=70^{\circ}$ [given]

$\angle BDC\Rightarrow\angle ADC=30^{\circ}$ [From (ii)]

$\angle ACD=\angle ACB+\angle BCD$

= 50° + 30° [From (i) and (ii)] = 80°

Now, $\angle ADC<\angle DAC<\angle ACD$

$AC<DC<AD$ [Side opposite to greater angle is longer and smaller angle is smaller]

$AD>CD$ and $AD>AC$

Hence proved

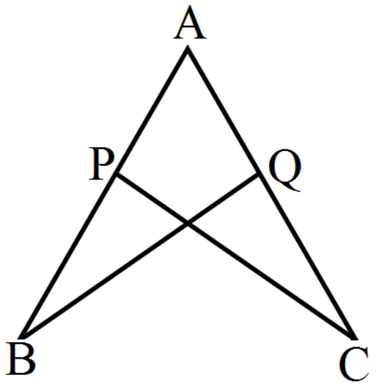
Or,

We have,

$\angle ACD > \angle DAC$ and $\angle ACD > \angle ADC$

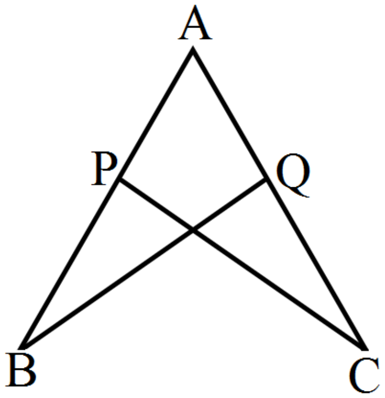
$AD > DC$ and $AD > AC$ [Side opposite to greater angle is longer and smaller angle is smaller]

118. In the given figure, if $AB = AC$ and $\angle B = \angle C$. Prove that $PQ = CP$.



Ans. : It is given that

$AB = AC$, and $\angle B = \angle C$



We have to prove that $BQ = CP$

We basically will prove $\triangle ABQ \cong \triangle ACP$ to show $BQ = CP$

In $\triangle ABQ$ and $\triangle ACP$

$\angle B = \angle C$ (Given)

$AB = AC$ (Given)

And $\angle A$ is common in both the triangles

So all the properties of congruence are satisfied

So $\triangle ABQ \cong \triangle ACP$

Hence $BQ = CP$ Proved.

119. In $\triangle ABC, \angle B = 35^\circ, \angle C = 65^\circ$ and the bisector of $\angle BAC$ meets BC in P. Arrange AP, BP and CP in descending order.

Ans. : In angle

A + B + C = 180

A + 35 + 65 = 180

A = 180 - 100

A = 80

So $\angle BAP$ and $\angle CAP = \frac{80}{2} = 40$

we know that side opposite to the greater angle is longer

In $\triangle \mathrm{BAP}$

we know that side opposite to the greater angle is longer

so BP > AP

In $\triangle \mathrm{CAP}$

ACP (65) > CAP (40)

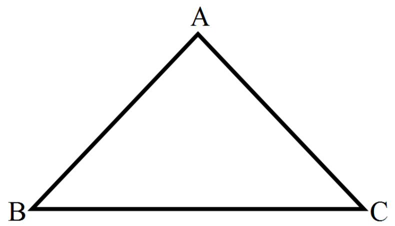
so AP > CP

so BP > AP > CP

120. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Ans. : Let $\triangle \mathrm{ABC}$ be isosceles such that $\mathrm{AB} = \mathrm{AC}$.

$\angle \mathrm{B} = \angle \mathrm{C}$



Given that vertex angle A is twice the sum of the base angles B and C. i..e.,

$\angle \mathrm{A} = 2(\angle \mathrm{B} = \angle \mathrm{C})$

$\angle \mathrm{A} = 2(\angle \mathrm{B} + \angle \mathrm{B})$

$\angle \mathrm{A} = 2(2\angle \mathrm{B})$

$\angle \mathrm{A} = 4(\angle \mathrm{B})$

Now, We know that sum of angles in a triangle = 180°

$\angle \mathrm{A} + \angle \mathrm{B} + \angle \mathrm{C} = 180^\circ$

$4\angle \mathrm{B} + \angle \mathrm{B} + \angle \mathrm{B} = 180^\circ$

$6\angle \mathrm{B} = 180^\circ$

$\angle \mathrm{B} = 30^\circ$

Since, $\angle \mathrm{B} = 4\angle \mathrm{B}$

$\angle \mathrm{B} = \angle \mathrm{C} = 30^\circ$

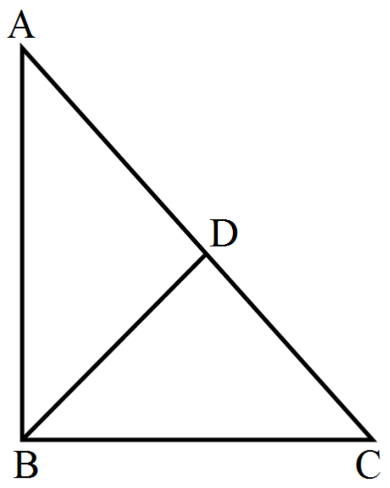
And $\angle \mathrm{A} = 4\angle \mathrm{B}$

$\angle \mathrm{A} = 4 \times 30^\circ = 120^\circ$

Therefore, angles of the given triangle are 120°, 30° and 30°.

$= 428$ and $\mathrm{LB} = \mathrm{LC}$

121. In a $\triangle \mathrm{ABC}, \mathrm{D}$ is the midpoint of side AC such that $\mathrm{BD} = \frac{1}{2}\mathrm{AC}$. Show that $\angle \mathrm{ABC}$ is a right angle.



Ans. :

D is the mid-point of AC.

$\Rightarrow \mathrm{AD} = \mathrm{CD} = \frac{1}{2}\mathrm{AC}$

Given, $\mathrm{BD} = \frac{1}{2}\mathrm{AC}$

$\Rightarrow \mathrm{AD} = \mathrm{CD} = \mathrm{BD}$

Consider $\mathrm{AD} = \mathrm{BD}$

$\Rightarrow \angle\mathrm{BAD} = \angle\mathrm{ABD} \ldots(\mathrm{i})$ (angles opposite to equal sides are equal)

Consider $\mathrm{CD} = \mathrm{BD}$

$\Rightarrow \angle\mathrm{BCD} = \angle\mathrm{CBD} \ldots(\mathrm{ii})$ (angles opposite to equal sides are equal)

In $\triangle\mathrm{ABC}$, angle sum property,

$\angle\mathrm{ABC} + \angle\mathrm{BAC} + \angle\mathrm{BCA} = 180^\circ$

$\Rightarrow \angle\mathrm{ABC} + \angle\mathrm{BAD} + \angle\mathrm{BCD} = 180^\circ$

$\Rightarrow \angle\mathrm{ABC} + \angle\mathrm{ABD} + \angle\mathrm{CBD} = 180^\circ$ [From (i) and (ii)]

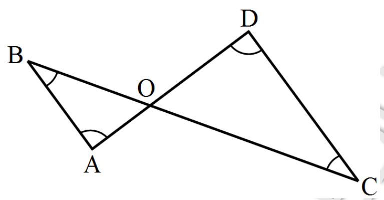
$\Rightarrow \angle\mathrm{ABC} + \angle\mathrm{ABC} = 180^\circ$

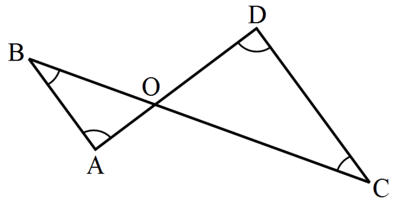
$\Rightarrow 2\angle\mathrm{ABC} = 180^\circ$

$\Rightarrow \angle\mathrm{ABC} = 90^\circ$

Hence, $\angle\mathrm{ABC}$ is a right angle.

122. In the given figure, $\angle\mathrm{B} < \angle\mathrm{A}$ and $\angle\mathrm{C} < \angle\mathrm{D}$. Show that $\mathrm{AD} < \mathrm{BC}$.





Ans. :

Given: $\angle\mathrm{B} < \angle\mathrm{A}$ and $\angle\mathrm{C} < \angle\mathrm{D}$

To prove: $\mathrm{AD} > \mathrm{BC}$

Proof:

In $\triangle\mathrm{AOB}$,

$\angle B < \angle A$

$\Rightarrow AO < BO$ (Side opposite to the greater angle is longer)...(1)

In $\triangle COD$,

$\angle C < \angle D$

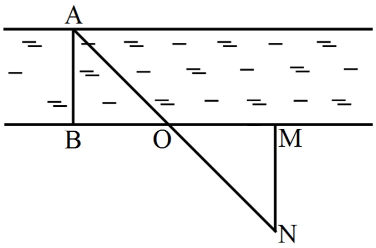
$\Rightarrow OD < OC$ (Side opposite to the greater angle is longer)...(2)

Adding (1) and (2), we get

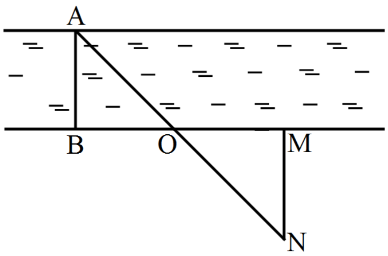
$AO + OD < BO + OC$

$\therefore AD < BC$

123. In the adjoining figure, explain how one can find the breadth of the river without crossing it.



Ans. : Let AB be the breadth of a river. Now take a point M on that bank of the river where point B is situated. Through M draw a perpendicular and take point N on it such that point, A, O and N lie on a straight line where point O is the mid point of BM.



Now in $\triangle ABO$ and $\triangle NMO$ we have,

$\angle OBA = \angle OMN = 90^\circ$

$OB = OM$ [$\because$ O is mid point of BM]

and $\angle BOA = \angle MON$ [Verticaly opposite angles]

Thus, by Angle-Side-Angle criterion of congruence, we have,

$\triangle ABO \cong \triangle NMO$ [By ASA]

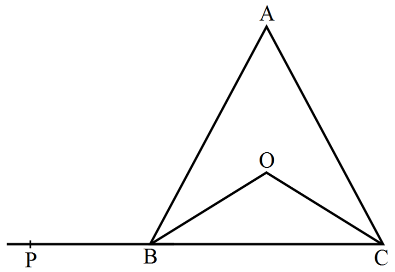
The corresponding parts of the congruent triangles are equal.

$\therefore AB = NM$ [C.P.C.T.]

Thus, we find that MN is the width of the river.

124. The bisectors of $\angle B$ and $\angle C$ of an isosceles $\triangle ABC$ with AB = AC intersect each other at a point O.Show that the exterior angle adjacent to $\angle ABC$ is equal to $\angle BOC$.

Ans. :



In $\triangle ABC$, $AB = AC$

$\Rightarrow \angle ABC = \angle ACB$

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle OBC = \angle OCB \ldots (i)$

In $\triangle BOC$, by angle sum property,

$\angle BOC + \angle OBC + \angle OCB = 180^\circ$

$\Rightarrow \angle BOC + 2\angle OBC = 180^\circ$ [from (i)]

$\Rightarrow \angle BOC + \angle ABC = 180^\circ$.

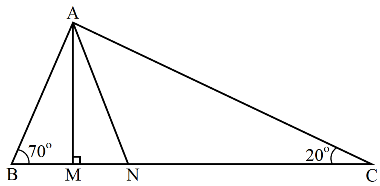
$\Rightarrow \angle BOC + (180^\circ - \angle ABP) = 180^\circ$ $\left(\triangle ABC \text{ and } \angle ABP \text{ form a linear pair}\right)$

$\Rightarrow \angle BOC + 180^\circ - \angle ABP = 180^\circ$

$\Rightarrow \angle BOC - \angle ABP = 0$

$\Rightarrow \angle BOC = \angle ABP$

125. In the given figure, $AM \perp BC$ and AN is the bisector of $\angle A$. If $\angle ABC = 70^\circ$ and $\angle ACB = 20^\circ$, find $\angle MAN$.



Ans. : In $\triangle ABC$, by angle sum property,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 70^\circ + 20^\circ = 180^\circ$

$\Rightarrow \angle A = 90^\circ$

In $\triangle ABM$, by angle sum property,

$\angle BAM + \angle ABM + \angle AMB = 180^\circ$

$\Rightarrow \angle BAM + 70^\circ + 90^\circ = 180^\circ$

$\Rightarrow \angle BAM = 20^\circ$

Since AN is the bisector of $\angle A$,

$\angle BAN = \frac{1}{2}\angle A$

$\Rightarrow \angle BAN = \frac{1}{2} \times 90^\circ = 45^\circ$

Now, $\angle MAN + \angle BAM = \angle BAN$

$\Rightarrow \angle MAN + 20^\circ = 45^\circ$

$\Rightarrow \angle MAN = 25^\circ$

126. In $\triangle PQR$, if $\angle P - \angle Q = 42^\circ$ and $\angle Q + \angle R = 21^\circ$, find $\angle P, \angle Q$ and $\angle R$.

Ans. : Since, $\angle P, \angle Q$ and $\angle R$ are the angles of a triangle.

So, $\angle P + \angle Q + \angle R = 180^\circ \ldots (i)$

Now, $\angle P - \angle Q = 42^\circ$ [Given]

$\Rightarrow \angle P = 42^\circ + \angle Q \ldots . (ii)$

and $\angle Q - \angle R = 21^\circ$ [Given]

$\angle R = \angle Q - 21^\circ \ldots . (iii)$

Substituting the value of $\angle P$ and $\angle R$ from (ii) and (iii) in (i), we get,

$42^\circ + \angle Q + \angle Q + \angle Q - 21^\circ = 180^\circ$

$\Rightarrow 3\angle Q + 21^\circ = 180^\circ$

$\Rightarrow 3\angle Q = 180^\circ - 21^\circ = 159^\circ$

$\Rightarrow \angle Q = \frac{159^\circ}{3} = 53^\circ$

$\therefore \angle P = 42^\circ + \angle Q$

$= 42^\circ + 53^\circ = 95^\circ$

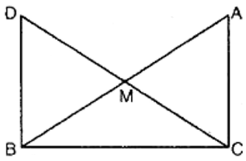
$\angle R = \angle Q - 21^\circ$

$= 53^\circ - 21^\circ = 32^\circ$

$\therefore \angle P = 95^\circ, \angle Q = 53^\circ$ and $\angle R = 32^\circ$.

* Answer the following questions. [5 Marks Each] [60]

127. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

i. $\triangle$AMC $\cong$ $\triangle$BMD
ii. $\angle$DBC is a right angle.
iii. $\triangle$DBC $\cong$ $\triangle$ACB
iv. CM = $\frac{1}{2}$ AB

Ans. :

i. In $\triangle$AMC and $\triangle$and BMD,
AM = BM [M is the mid-point of AB]
$\angle$AMC =$\angle$ BMD [Vertically opposite angles]
CM = DM [Given]
$\therefore$ $\triangle$AMC $\cong$ $\triangle$BMD [By SAS congruency]
$\therefore$ $\angle$ ACM =$\angle$ BDM ...(i)

ii. For two lines AC and DB and transversal DC, we have,
$\angle$ACD = $\angle$BDC [Alternate angles]
$\therefore$ AC $\parallel$ DB
Now for parallel lines AC and DB and for transversal BC.
$\angle$ DBC = $\angle$ ACB [Alternate angles] ...(ii)
ButΔ ABC is a right angled triangle, right angled at C.

$\therefore \angle ACB = 90°$ (iii)

Therefore $\angle$ DBC = 90° [Using eq. (ii) and (iii)]

$\Rightarrow \angle$ DBC is a right angle.

iii. Now in $\triangle$DBC and $\triangle$ ABC,

DB = AC [Proved in part (i)]

$\angle$ DBC = $\angle$ACB = 90° [Proved in part (ii)]

BC = BC [Common]

$\therefore \triangle DBC \cong \triangle ACB$ [By SAS congruency]

iv. Since $\triangle DBC \cong \triangle ACB$ [Proved above]

$\therefore$ DC = AB

$\Rightarrow$ AM + CM = AB

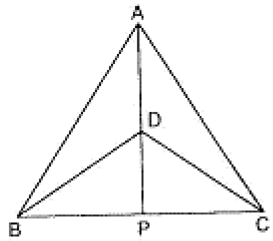
$\Rightarrow$ CM + CM = AB [$\because$ DM = CM]

$\Rightarrow$ 2CM = AB

$\Rightarrow$ CM = $\frac{1}{2}$ AB

128. $\triangle$ABC and $\triangle$DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that :

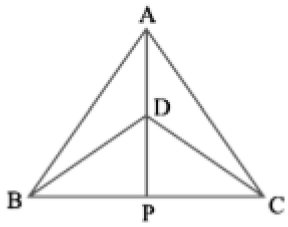
i. $\triangle ABD \cong \triangle ACD$
ii. $\triangle ABP \cong \triangle ACP$
iii. AP bisects $\angle$A as well as $\angle$D
iv. AP is the perpendicular bisector of BC.



Ans. :

It is given in the question that:

$\triangle$ ABC and $\triangle$DBC are two isosceles triangles



i. $\triangle$ ABC and $\triangle$DBC

AD = AD (Common)

AB = AC (Triangle ABC is isosceles)

BD = CD (Triangle DBC is isosceles)

By SSS axiom,

$\triangle ABD \cong \triangle ACD$

ii. In $\triangle$ ABP and $\triangle$ACP,

AP = AP (Common)

$\angle$PAB = $\angle$PAC (By c.p.c.t)

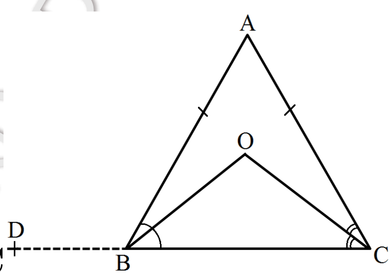
AB = AC (Triangle DBC is isoceles)
By SAS axiom,
$\triangle ABP \cong \triangle ACP$

iii. $\angle$PAB = $\angle$PAC { c.p.c.t }
AP bisects $\angle$A (i)
Also,
In $\triangle$ BPD and $\triangle$CPD,
PE = PD (Common)
BD = CD (Triangle DBC is isosceles)
BP = CP ($\triangle ABP \cong \triangle ACP$ so by c.p.c.t)
By SSS axiom,
$\triangle BPD \cong \triangle CPD$
Thus,
$\angle$BDP = $\angle$CDP { c.p.c.t } (ii)
By (i) and (ii), we can say that $\angle$AP bisects $\angle$A as well as D

iv. $\angle$BPD = $\angle$CPD (By c.p.c.t)
And,
BP = CP (i)
Also,
$\angle$BPD = $\angle$CPD = 180° (BC is a straight line)
2$\angle$BPD = 180°
$\angle$BPD = 90° (ii)
From (i) and (ii), we get
AP is the perpendicular bisector of BC

129. Bisectors of the angles B and C of an isosceles $\triangle \mathrm{ABC}$ with AB = AC intersect each other at O. Show that external angle adjacent to $\angle \mathrm{ABC}$ is equal to $\angle \mathrm{BOC}$.

Ans. : Given Lines, BO and CO are angle bisectore isosceles such that AB = AC which $\angle \mathrm{ABC}$ and $\angle \mathrm{ACB}$ respectively at O.
$\angle \mathrm{DBA} = \angle \mathrm{BOC}$
In $\triangle \mathrm{ABC}$,
$\mathrm{AB} = \mathrm{AC}$
$\angle \mathrm{ACB} = \angle \mathrm{ABC}$
$\Rightarrow \frac{1}{2}\angle \mathrm{ACB} = \frac{1}{2}\angle \mathrm{ABC}$

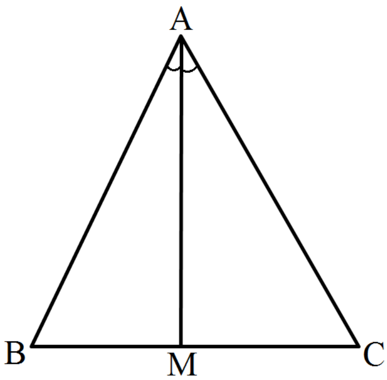


$\Rightarrow \angle \mathrm{OCB} = \angle \mathrm{OBC}$ In $\triangle \mathrm{OCB}$,
$\angle \mathrm{OCB} + \angle \mathrm{OCB} + \angle \mathrm{BOC} = 180^\circ$
$\Rightarrow \angle \mathrm{OBC} + \angle \mathrm{OBC} + \angle \mathrm{BOC} = 180^\circ$
$\Rightarrow 2\angle \mathrm{OBC} + \angle \mathrm{BOC} = 180^\circ$
$\Rightarrow \angle \mathrm{ABC} + \angle \mathrm{BOC} = 180^\circ$

$\Rightarrow 180^\circ - \angle \mathrm{DBA} + \angle \mathrm{BOC} = 180^\circ$
$\Rightarrow \angle \mathrm{DBA} + \angle \mathrm{BOC} = 0$
$\Rightarrow \angle \mathrm{DBA} = \angle \mathrm{BOC}$

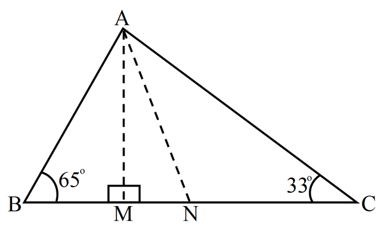
130. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle \mathrm{BCA}$. Is it true to say that perimeter of the triangle is greater than 2AM? Give reason for your answer?

Ans. : In $\triangle \mathrm{ABC}$, M is point of side BC such that AM is the bisector
$\mathrm{AB} + \mathrm{BM} > \mathrm{AM} \ldots. \text{(i)}$
In $\triangle \mathrm{ACM}$, we have



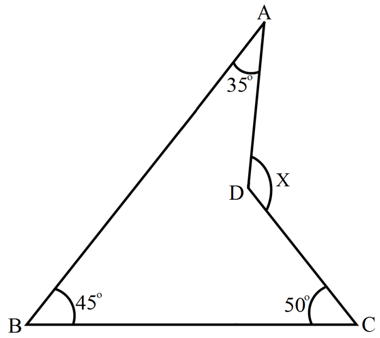
$\mathrm{AC} + \mathrm{CM} > \mathrm{AM} \ldots \text{(ii)}$ On adding eq.(i) and (ii),
$\Rightarrow (\mathrm{AB} + \mathrm{BM} + \mathrm{AC} + \mathrm{CM}) > 2\mathrm{AM}$
$\Rightarrow (\mathrm{AB} + \mathrm{BM} + \mathrm{CM} + \mathrm{AC}) > 2\mathrm{AM}$
$\Rightarrow \mathrm{AB} + \mathrm{BC} + \mathrm{AC} > 2\mathrm{AM}$

131. In Fig. $\mathrm{AM} \perp \mathrm{BC}$ and AN is the bisector of $\angle \mathrm{A}$. If $\angle \mathrm{B} = 65^\circ$ and $\angle \mathrm{C} = 33^\circ$, find $\angle \mathrm{MAN}$.



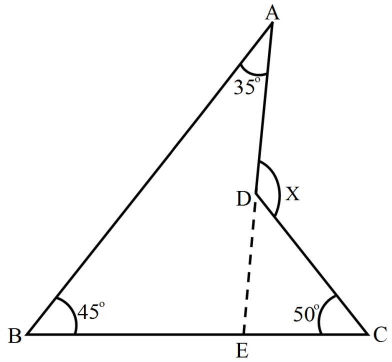
Ans. : Let $\angle \mathrm{BAN} = \angle \mathrm{NAC} = \mathrm{x}\ [\because \mathrm{AN} \text{ bisects } \angle \mathrm{A}]$
$\therefore \angle \mathrm{ANM} = \mathrm{x} + 33^\circ$ [Exterior angle property]
In $\triangle \mathrm{AMB}$
$\angle \mathrm{BAM} = 90^\circ - 65^\circ = 25^\circ$ [Exterior angle property]
$\therefore \angle \mathrm{MAN} = \angle \mathrm{BAN} - \angle \mathrm{BAM} = (\mathrm{x} - 25)^\circ$
Now in $\triangle \mathrm{MAN}$,
$(\mathrm{x} - 25)^\circ + (\mathrm{x} + 33)^\circ + 90^\circ = 180^\circ$ [Angle sum property]
$\Rightarrow 2\mathrm{x} + 8^\circ = 90^\circ$
$\Rightarrow 2\mathrm{x} = 82^\circ$
$\Rightarrow \mathrm{x} = 41^\circ$
$\therefore \mathrm{MAN} = \mathrm{x} - 25^\circ$
$= 41^\circ - 25^\circ$
$= 16^\circ$

132. In the given figure, compute the value of x.



Ans. : In the given figure, $\angle \mathrm{DCB} = 45^\circ, \angle \mathrm{CBA} = 35^\circ$ and $\angle \mathrm{BAD} = 35^\circ$

Here, we will produce AD to meet BC at E



Now, using angle sum property of the triangle

In $\triangle \mathrm{AEB}$

$\angle \mathrm{BAE} + \angle \mathrm{AEB} + \angle \mathrm{EBA} = 180^\circ$

$\angle \mathrm{AEB} + 35^\circ + 45^\circ = 180^\circ$

$\angle \mathrm{AEB} + 180^\circ = 180^\circ$

$\angle \mathrm{AEB} = 180^\circ - 80^\circ$

$\angle \mathrm{AEB} = 100^\circ$

Eurther, BEC is a straight line. So, using the property, "the angles forming a linear pair are supplementary", we get

$\angle \mathrm{AEB} + \angle \mathrm{AEC} = 180^\circ$

$100^\circ + \angle \mathrm{AEC} = 180^\circ$

$\angle \mathrm{AEC} = 180^\circ - 100^\circ$

$\angle \mathrm{AEC} = 80^\circ$

Also, using the property, " an exterior angle of a triangle is equal to the sum of its two opposite interior angles"

In $\triangle \mathrm{DEC}, \mathrm{x}$ is its exterior angle

Thus,

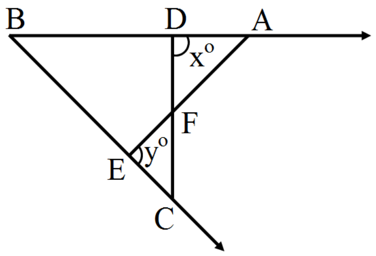
$\angle \mathrm{x} = \angle \mathrm{DCE} + \angle \mathrm{DEC}$

$= 50^\circ + 80^\circ$

$= 130^\circ$

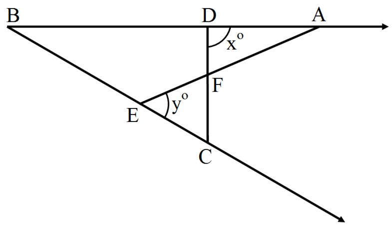
Therefore, $\mathrm{x} = 130^\circ$.

133. In the given figure, if x = y and AB = CB then prove that AE = CD.



Ans. : Given: AB = BC

and, x° = y°



To prove: AE = CD

Proof: In $\triangle \mathrm{ABE}$, we have,

Exterior $\angle \mathrm{AEB} = \angle \mathrm{EBA} + \angle \mathrm{BAE}$

$\Rightarrow \mathrm{y}^{\circ} = \angle \mathrm{EBA} + \angle \mathrm{BAE}$

Again, in $\triangle \mathrm{BCD}$ we have

$\mathrm{x}^{\circ} = \angle \mathrm{CBA} + \angle \mathrm{BCD}$

Since, x = y [Given]

SO, $\angle \mathrm{EBA} + \angle \mathrm{BAE} = \angle \mathrm{CBA} + \angle \mathrm{BCD}$

$\Rightarrow \angle \mathrm{BAE} = \angle \mathrm{BCD}$

Thus in $\triangle \mathrm{BCD}$ and $\triangle \mathrm{BAE}$, we have

$\angle \mathrm{B} = \angle \mathrm{B}$ [Common]

$\mathrm{BC} = \mathrm{AB}$ [Given]

and, $\angle \mathrm{BCD} = \angle \mathrm{BAE}$ [Proved above]

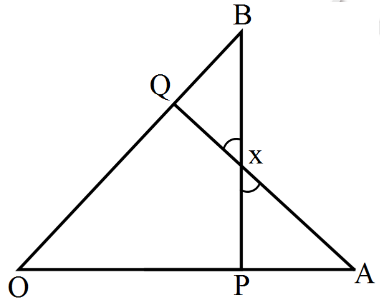
Thus by Angle-Side-Angle criterion of congruence, we have

$\triangle \mathrm{BCD} \cong \triangle \mathrm{BAE}$

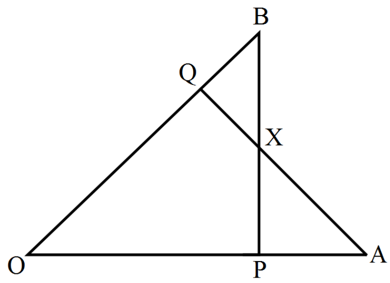
The corresponding parts of the congruent triangles are equal.

So, $\mathrm{CD} = \mathrm{AE}$ [Proved]

134. In the given figure, OA = OB and OP = OQ. Prove that (i) PX = QX, (ii) AX = BX.



Ans. : Given: $\mathrm{OA} = \mathrm{OB}$ and $\mathrm{OP} = \mathrm{OQ}$



To Prove:

i. PX = QX
ii. AX = BX

Proof: In $\triangle OAQ$ and $\triangle OPB$, we have,

$OA = OB$ [Given]

$\angle O = \angle O$ [Common]

$OQ = OP$ [given]

Thus by Side-angle-side criterion of congruence, we have

$\triangle OAQ = \triangle OPB$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore \angle OBP = \angle OAQ \ldots (1)$

Thus, in $\triangle BXQ$ and $\triangle PXA$, we have

$BQ = OB - OQ$

and, $PA = OA - OP$

But, $OP = OQ$

and $OA = OB$ [Given]

Therefore, we have, $BQ = PA \ldots (2)$

Now consider triangles $\triangle BXQ$ and $\triangle PXA$.

$\angle BXQ = \angle PXA$ [Vertical opposite angles]

$\angle OBP = \angle OAQ$ [from (1)]

$BQ = PA$ [from (2)]

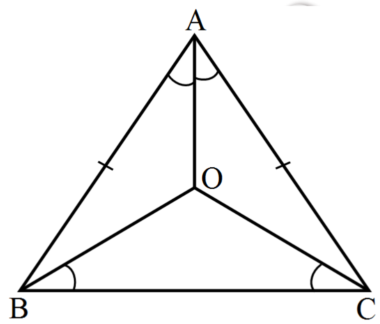
Thus by Angle-Angle-Side criterion of congruence, we have,

$\triangle BXQ \cong \triangle PXA$

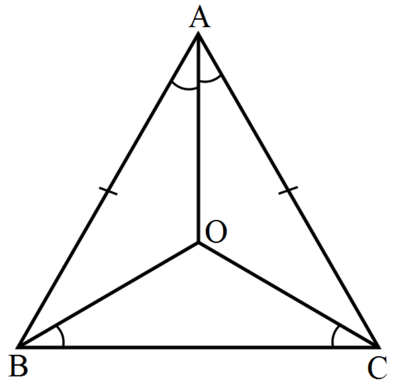
$PX = QX$ [C.P.C.T.]

$AX = BX$ [C.P.C.T.]

135. In $\triangle ABC$, AB = AC and the bisectors of $\angle B$ and $\angle C$ meet at a point O. Prove that BO = CO and the ray AO is the bisector of $\angle A$.



Ans. : Given: A $\triangle ABC$ in which AB = AC, BO and CO are bisectors of $\angle B$ and $\angle C$



To Prove: In $\triangle\mathrm{BOC}$, we have,

$\angle\mathrm{OBC} = \frac{1}{2}\angle\mathrm{B}$

and, $\angle\mathrm{OBC} = \frac{1}{2}\angle\mathrm{C}$

But, $\angle\mathrm{B} = \angle\mathrm{C}$ [$\therefore$ AB = AC (given)]

SO, $\angle\mathrm{OBC} = \angle\mathrm{OCB}$

Since base angle are equal, sides are equal

$\Rightarrow \mathrm{OB} = \mathrm{OC}\ldots(1)$

Since OB and OC are the bisectors of angles, $\angle\mathrm{B}$ and $\angle\mathrm{C}$ respectively, we have

$\angle\mathrm{ABO} = \frac{1}{2}\angle\mathrm{B}$

$\angle\mathrm{ACO} = \frac{1}{2}\angle\mathrm{C}$

$\Rightarrow \angle\mathrm{ABO} = \angle\mathrm{ACO}\ldots(2)$

Now, in $\triangle\mathrm{ABO}$ and $\triangle\mathrm{ACO}$

$\mathrm{AB} = \mathrm{AC}$ [Given]

$\angle\mathrm{ABO} = \angle\mathrm{ACO}$ [from (2)]

$\mathrm{BO} = \mathrm{OC}$ [from (1)]

Thus, by Side-Angle-side criterion of congruence, we have

$\triangle\mathrm{ABO} \cong \triangle\mathrm{ACO}$ [BY SAS]

The corresponding parts of the congruent triangles are equal

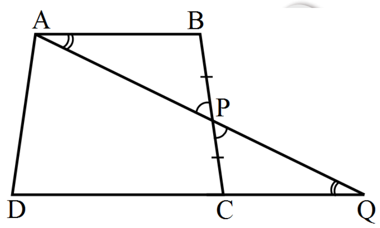
$\therefore \angle\mathrm{BAO} = \angle\mathrm{CAO}$ [By C.P.C.T.]

i.e. AO bisects $\angle\mathrm{A}$.

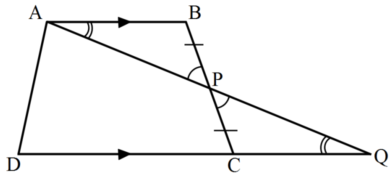
136. In the given figure, ABCD is a quadrilateal in which AB || DC and P is the midpoint of BC. On producing, AP and DC meet at Q. Prove that

i. AB = CQ,

ii. DQ = DC + AB.



Ans. :



Given ABCD is a quadrilateral in which AB || DC

To Prove:

i. AB = CQ

ii. DQ = DC + AB

Proof: In $\triangle \mathrm{ABP}$ and $\triangle \mathrm{PCQ}$ we have

$\angle \mathrm{PAB}=\angle \mathrm{PQC}$ [alternate angles]

$\angle \mathrm{APB}=\angle \mathrm{CPQ}$ [Vertically opposite angles]

$\mathrm{BP}=\mathrm{PC}$ [Given]

Thus by Angle-Angle-Side criterion of congruence, we have

$\triangle \mathrm{ABP} \cong \triangle \mathrm{PCQ}$

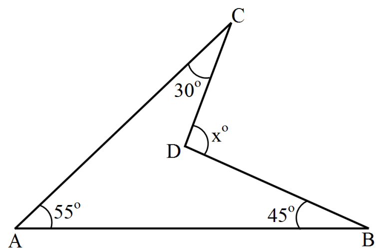
The corresponding parts of the congruents triangles are equal

$\therefore \mathrm{AB}=\mathrm{CQ} \ldots(1)$

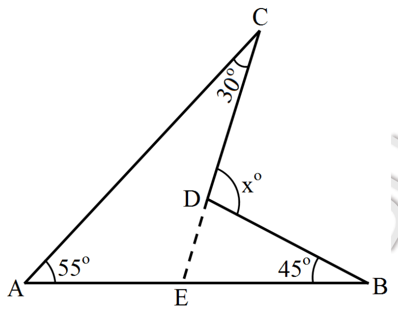
Now, $\mathrm{DQ}=\mathrm{DC}+\mathrm{CQ}$

$=\mathrm{DC}+\mathrm{AB}$ [from (1)]

137. Calculate the value of x in the given figure.



Ans. : Produce CD to cut AB at E.



Now, in $\triangle \mathrm{BDE}$, we have,

Exterior $\angle" \mathrm{CDB}=\angle \mathrm{CEB}+\angle \mathrm{DBE}$

$\Rightarrow \mathrm{x}^{\circ}=\angle \mathrm{CEB}+45^{\circ} \ldots .(\mathrm{i})$

In $\triangle \mathrm{AEC}$ we have,

Exterior $\angle \mathrm{CEB}=\angle \mathrm{CAB}+\angle \mathrm{ACE}$

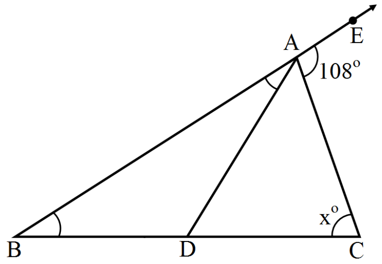
$55^{\circ}+30^{\circ}=85^{\circ}$

Putting $\angle \mathrm{CEB}=85^{\circ}$ in (i), we get,

$\mathrm{x}^{\circ}=85^{\circ}+45^{\circ}=130^{\circ}$

$\therefore x = 130$

138. In the given figure, AD divides $\angle BAC$ in the ratio 1 : 3 and AB = DB. Determine the value of x.



Ans. : The angle $\angle BAC$ is divided by AD in the ratio 1 : 3.

Let $\angle BAD$ and $\angle DAC$ be y and 3y, respectively.

As BAE is a straight line,

$\angle BAC = \angle CAE = 180^\circ$ [linear pair]

$\Rightarrow \angle BAD + \angle DAC + \angle CAE = 180^\circ$

$\Rightarrow y + 3y + 108^\circ = 180^\circ$

$\Rightarrow 4y = 180^\circ - 108^\circ = 72^\circ$

$\Rightarrow y = \frac{72^\circ}{4} = 18^\circ$

Now, in $\triangle ABC,$

$\angle ABC + \angle BCA + \angle BAC = 180^\circ$

$y + x + 4y = 180^\circ$

$[$ Since, $\angle ABC = \angle BAD$ (given AD = DB) and $\angle BAC = y + 3y = 4y]$

$\Rightarrow 5y + x = 180$

$\Rightarrow 5 \times 18 + x = 180$

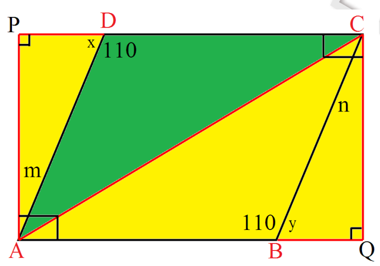
$\Rightarrow 90 + x = 180$

$\Rightarrow x = 180 - 90 = 90$

* **Case study based questions.** **[12]**

139. Read the Source/ Text given below and answer these questions:

In the middle of the city, there was a park ABCD in the form of a parallelogram form so that AB = CD, AB || CD and AD = BC, AD || BC Municipality converted this park into a rectangular form by adding land in the form of $\triangle APD$ and $\triangle BCQ$. Both the triangular shape of land were covered by planting flower plants.



Answer the following questions:

i. What is the value of $\angle x$?

a. 110°
b. 70°
c. 90°

d. 100°

ii. $\triangle \mathrm{APD}$ and $\triangle \mathrm{BCQ}$ are congruent by which criteria?

a. SSS
b. SAS
c. ASA
d. RHS

iii. PD is equal to which side?

a. DC
b. AB
c. BC
d. BQ

iv. $\triangle \mathrm{ABC}$ and $\triangle \mathrm{ACD}$ are congruent by which criteria?

a. SSS
b. SAS
c. ASA
d. RHS

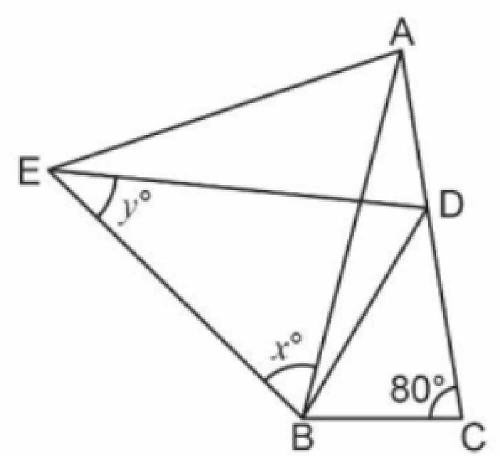
v. What is the value of $\angle \mathrm{m}$?

a. 110°
b. 70°
c. 90°
d. 20°

Ans. :

(i)	(b)	70°
(ii)	(c)	ASA
(iii)	(d)	BQ
(iv)	(a)	SSS
(v)	(d)	20°

140. In the given figure, the isosceles triangle ABC ≅ EAD. The point E is equidistant from both A and B.



4. What is the value of x?
A. 40°
B. 60°
C. 70°
D. 80°

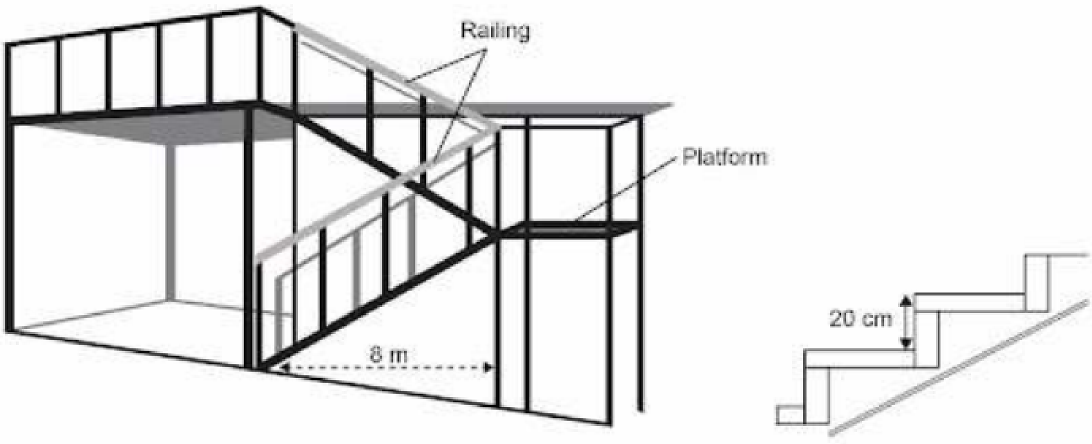
5. What is the value of y?

6. What is the value of ∠BDC?

A. 30°
B. 40°
C. 50°
D. 70°

Ans. : 4. B. 60°
5. 40
40°
6. A. 30°

141. The picture below shows a staircase outside a house. Each step of the staircase is congruent and there are 25 steps in the staircase from the loor to the platform and 25 steps from the platform to the roof.



7. What is the length of the staircase railing?

Ans. : 7. $2\sqrt{89}\,m$

----- “काक: चेष्टा, बको ध्यानं, श्वान निद्रा तथैव च। अल्पाहारी, गृहत्यागी, विद्यार्थी पंचलक्षणम् ॥” -----